

## *Evaluation of closed-form analytical models for predicting unsaturated soil hydraulic properties*

Yuji TAKESHITA\* and Ichiro KOHNO\*\*

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### **SYNOPSIS**

Knowledge of the unsaturated soil hydraulic properties is essential requirement for prediction of seepage flow and contaminant transport through the vadose zone. Unfortunately, these parameters are usually time consuming and expensive to measure in the field and laboratory. At the present condition, there are few data accumulation for Japanese soils. In this paper, van Genuchten's closed-form expressions are described to estimate unsaturated soil hydraulic properties. To evaluate the adequacy of these expressions, comparisons are performed between observed and calculated unsaturated hydraulic properties for typical Japanese soils.

### **1. INTRODUCTION**

Knowledge of the unsaturated soil hydraulic properties is essential requirement for prediction of seepage flow and contaminant transport through the vadose zone. The unsaturated soil hydraulic properties consist of the hydraulic conductivity as a function of pressure head and the soil water retention curve as a relationship between negative pressure head and volumetric water content. Fig.1 is the explanation drawing of these relations. Unsaturated hydraulic conductivity is one of the most important hydraulic properties governing the saturated-unsaturated transport of fluid and solutes in soil. Unfortunately, unsaturated hydraulic parameters are usually time consuming and expensive to measure in the field and laboratory. At the present condition, there are few data accumulation for Japanese soils. Therefore, many

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\* The Graduate School of Natural Science and Technology

\*\* Department of Civil Engineering

methods have been developed to estimate unsaturated properties from empirical equations or calculate them from measurements of water retention curve<sup>(1,2,3,4)</sup>. It is the purpose of this paper to apply van Genuchten's closed-form expressions<sup>(5)</sup> to several water retention data which are measured in Japan and to evaluate the feasibility of determining the unsaturated soil hydraulic properties by Van Genuchten's computer code<sup>(6)</sup>.

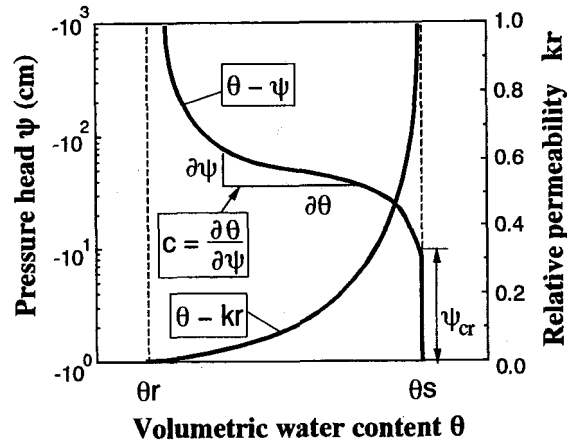


Fig.1 Unsaturated soil hydraulic properties

## 2. PARAMETRIC MODEL FOR THE UNSATURATED SOIL HYDRAULIC PROPERTIES

The unsaturated soil hydraulic properties are strongly nonlinear functions of the negative pressure head. It is assumed that suitable analytical expressions for these functions are available. Van Genuchten proposed a closed-form analytical solution to the theoretical models developed by Burdine<sup>(7)</sup> and Mualem<sup>(8)</sup>.

$$S_e = (\theta - \theta_r) / (\theta_s - \theta_r) = \{ 1 + |\alpha \psi|^n \}^{-m} \quad (1)$$

$$k(\psi) = k_s \cdot S_e^{1/2} \{ 1 - (1 - S_e^{1/m})^m \}^2 \quad (2)$$

$$C(\psi) = \alpha(n-1)(\theta_s - \theta_r) S_e^{1/m} (1 - S_e^{1/m})^m \quad (3)$$

where  $m=1-1/n$  ( $0 < m < 1, n > 1$ ),  $S_e$  is the effective saturation,  $\theta_s$  is the saturated water content,  $\theta_r$  is the

residual water content,  $k_s$  is the saturated conductivity, and  $\alpha$ ,  $n$  are the soil water retention curve shape parameters ( empirical parameters).  $k(\psi)$  is the hydraulic conductivity as a function of the negative pressure head  $h$ .  $C(\psi)$  is the soil water capacity, being the slope  $d\theta/dh$  of the soil water retention curve. From Eq.(1) through Eq.(3), there are five parameters  $k_s$ ,  $\theta_s$ ,  $\theta_r$ ,  $\alpha$  and  $n$  in these expressions, the first two have clear physical significance and are independently measured from laboratory tests. The residual water content is defined nominally as the water content at which  $k$  is zero and  $h$  is negative infinitely great. It is considered that the residual water content for sandy soil is equal to zero. Respectively, the parameter  $\alpha$  and  $n$  are inversely related to the air-entry value and width of the pore distribution. From our own data,  $\alpha$  generally ranges from 0.02 to 0.1 [1/cm], while  $n$  usually varies from 3 to 10 for sandy soils.

Eq.(1) is valid for monotonic wetting or drying only. When the unsaturated flow process involves both wetting and drying, hysteresis in the  $C(h)$  relation will have to be taken account as Figure 2 shows. But in this paper, effect of hysteresis is omitted as it is not so important to evaluate the average unsaturated soil hydraulic properties.

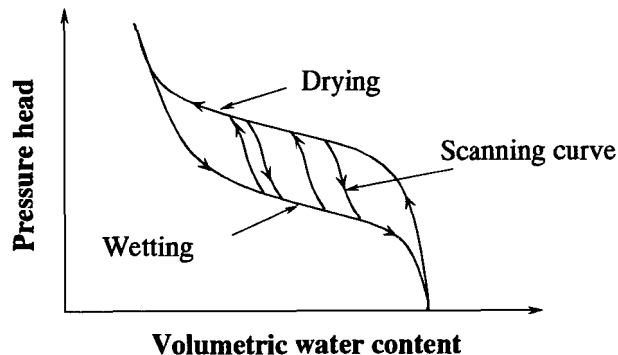


Fig.2 Hysteresis of soil water retention curve

### 3. PARAMETER ESTIMATION PROCEDURE

Van Genuchten describes a graphical procedure to estimate  $\alpha$  and  $n$  from soil water retention data, and indicates that  $\theta_s$  is easily obtained in laboratory experiments. Van Genuchten also developed a computer code to estimate  $\alpha$  and  $n$  by non-linear least-squares regression procedure; for  $\theta_r$  is unknown, the regression code also estimates  $\theta_r$ . So, this code can compute three parameters  $\alpha$ ,  $n$ , and  $\theta_r$  for the Mualem and Burdine models.

This approach is very simple and convenient, but there are some disadvantages in the procedure as,

1) it need soil water retention data collection, but the data are generally time-consuming.

2) the fact that parameters are fitted to water retention data only, so that any inaccuracy in the assumed hydraulic relationships, as well as effects of measurement error are forced into the predicted unsaturated hydraulic conductivity as a function of the negative pressure head<sup>(9)</sup>.

#### 4. APPLICATION TO OBSERVED DATA

In this section, Comparisons are given between observed and calculated unsaturated hydraulic properties for six soil. These are typical examples in Japan<sup>(10,11,12,13,14,15)</sup>. Table 1 summarizes some of the soil-physical properties of six soils. Estimates of the parameters  $\alpha$ ,  $n$ , and  $\theta_r$  are also include in this table. Soil water retention data of these samples were measured by the tensiometer method or the instantaneous profile method. Van Genuchten's code are applied to the soil water retention data. In this approach, saturated volumetric water content are set to porosity. Comparison of measured unsaturated soil hydraulic properties and estimated one by Van Genuchten's expressions are shown in Fig.3 - Fig.8. As seen in these figures, a reasonable correspondence is obtained between estimated data and measured one.

Table 1 Soil-physical properties of six soils

Fig. No.	Sample name	Estimated parameters			Known parameters		Reference
		$\alpha$ (1/cm)	$n$	$\theta_r$	$k_s$ (cm/s)	$\theta_s$	
3	Fine sand	0.0356	4.793	0.042	$2.864 \times 10^{-2}$	0.403	(10)
4	Kanto Loam	0.0115	1.487	0.218	$4.500 \times 10^{-3}$	0.760	(11)
5	Shirasu A	0.0286	4.78	0.294	$2.254 \times 10^{-4}$	0.520	(12)
	Shirasu B	0.0167	3.116	0.128	$1.000 \times 10^{-4}$	0.600	
6	Mud stone	0.0759	1.455	0.021	$1.700 \times 10^{-7}$	0.580	(13)
7	Toyoura Sand	0.0218	12.318	0.00	$2.560 \times 10^{-2}$	0.411	(14)
8	Granite soil A	0.111	2.08	0.075	$9.000 \times 10^{-3}$	0.348	(15)
	Granite soil B	0.102	2.13	0.199	$3.100 \times 10^{-2}$	0.517	
	Granite soil C	0.0349	2.06	0.270	$6.200 \times 10^{-3}$	0.554	

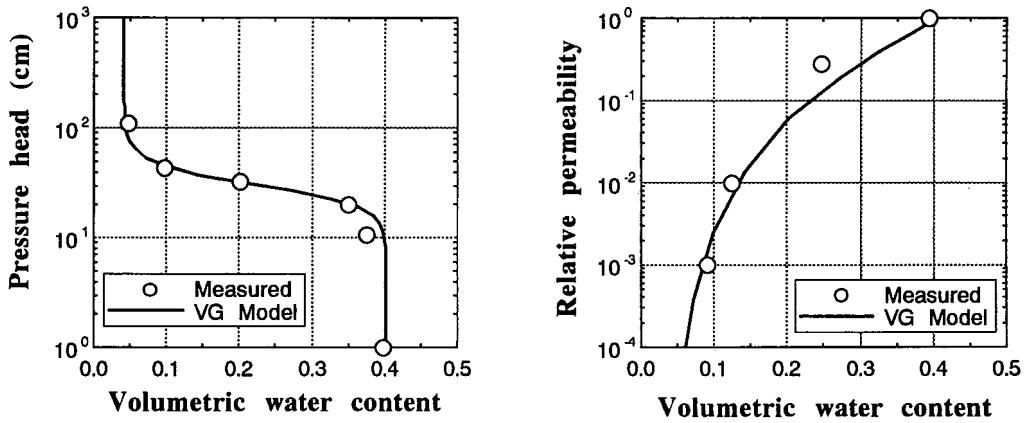


Fig. 3 Observed and calculated data of the soil hydraulic properties of Fine sand

$$(\theta_s=0.403, k_s=2.864 \times 10^{-2} \text{ cm/s}, \theta_r=0.042, \alpha=0.0356 \text{ cm}^{-1}, n=4.793)$$

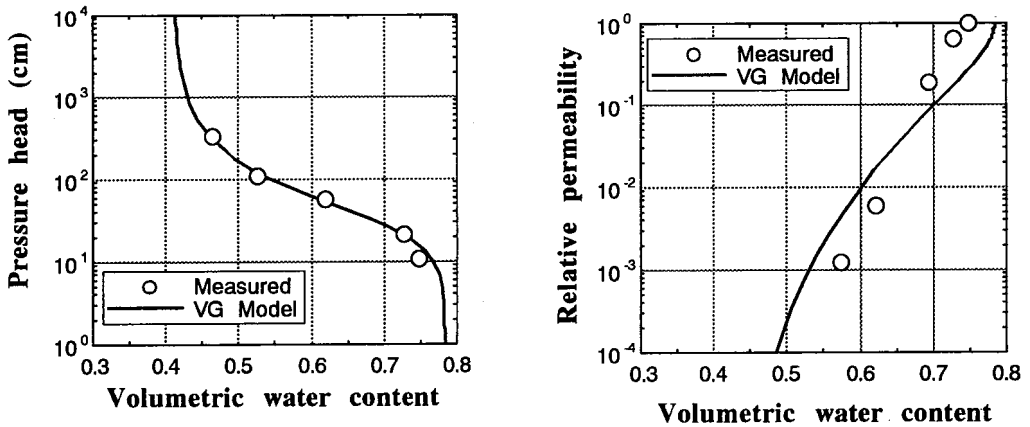


Fig. 4 Observed and calculated data of the soil hydraulic properties of Kanto Loam

$$(\theta_s=0.760, k_s=4.500 \times 10^{-3} \text{ cm/s}, \theta_r=0.218, \alpha=0.0115 \text{ cm}^{-1}, n=1.487)$$

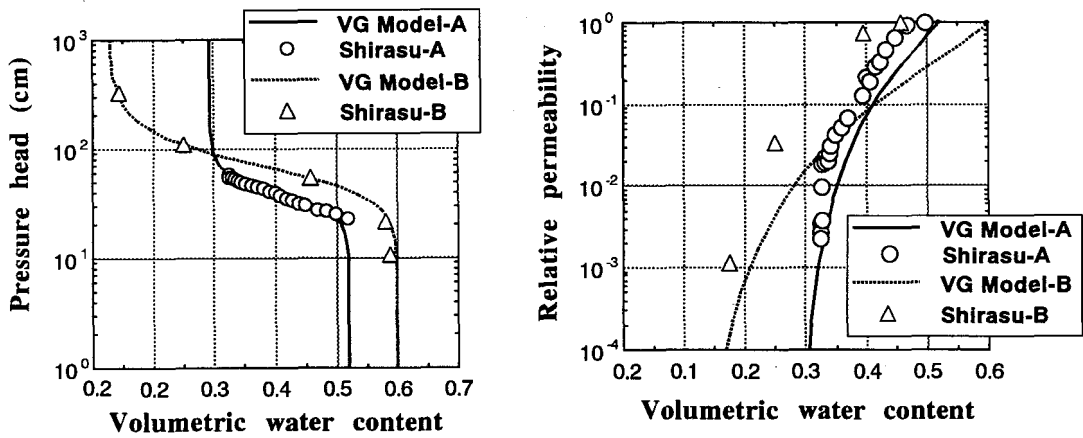


Fig. 5 Observed and calculated data of the soil hydraulic properties of Shirasu

(Shirasu A:  $\theta_s=0.520$ ,  $k_s=2.254 \times 10^{-4}$  cm/s,  $\theta_r=0.294$ ,  $\alpha=0.0286$  cm $^{-1}$ ,  $n=4.780$ )

(Shirasu B:  $\theta_s=0.600$ ,  $k_s=1.000 \times 10^{-4}$  cm/s,  $\theta_r=0.128$ ,  $\alpha=0.0167$  cm $^{-1}$ ,  $n=3.116$ )

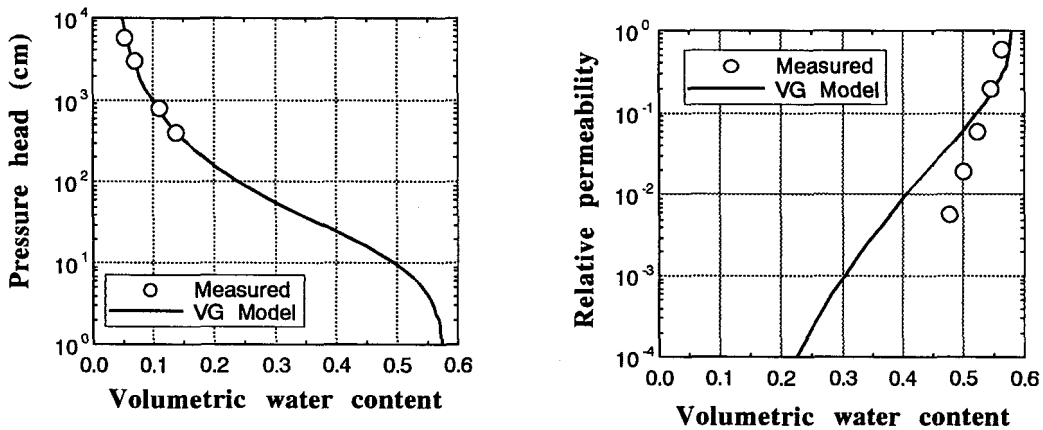


Fig. 6 Observed and calculated data of the soil hydraulic properties of Mud stone

( $\theta_s=0.580$ ,  $k_s=1.700 \times 10^{-7}$  cm/s,  $\theta_r=0.021$ ,  $\alpha=0.0759$  cm $^{-1}$ ,  $n=1.455$ )

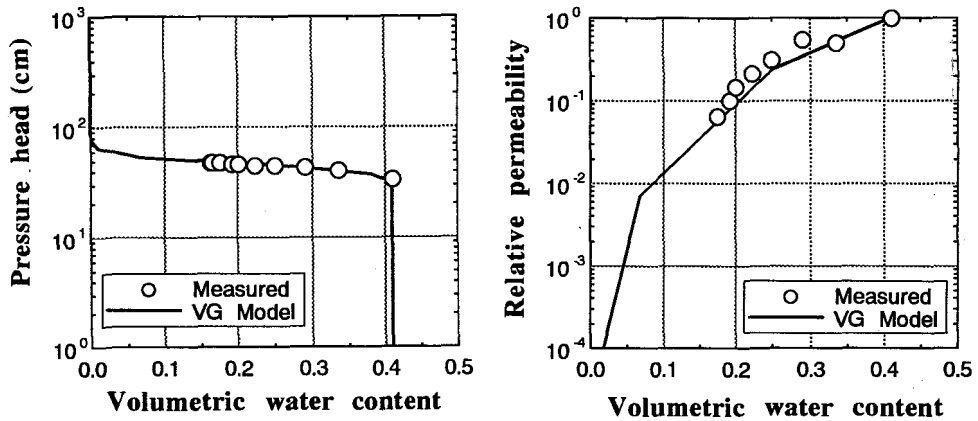


Fig. 7 Observed and calculated data of the soil hydraulic properties of Toyoura sand (Fine sand)

$$(\theta_s=0.411, k_s=2.560 \times 10^{-2} \text{ cm/s}, \theta_r=0.00, \alpha=0.0218 \text{ cm}^{-1}, n=12.318)$$

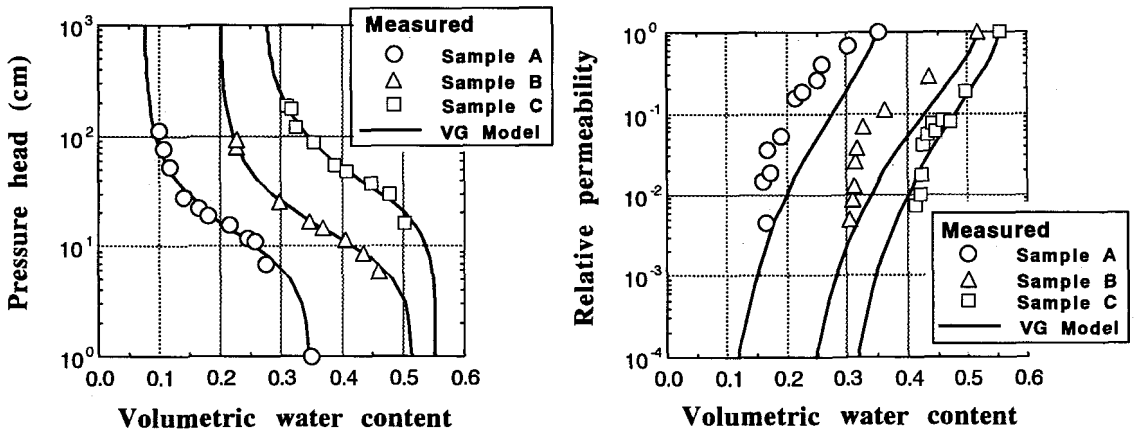


Fig. 8 Observed and calculated data of the soil hydraulic properties of Granite soil

$$(\text{Sample A: } \theta_s=0.348, k_s=9.0 \times 10^{-3} \text{ cm/s}, \theta_r=0.075, \alpha=0.111 \text{ cm}^{-1}, n=2.08)$$

$$(\text{Sample B: } \theta_s=0.517, k_s=3.1 \times 10^{-3} \text{ cm/s}, \theta_r=0.199, \alpha=0.102 \text{ cm}^{-1}, n=2.13)$$

$$(\text{Sample C: } \theta_s=0.554, k_s=6.2 \times 10^{-3} \text{ cm/s}, \theta_r=0.27, \alpha=0.0349 \text{ cm}^{-1}, n=2.06)$$

## 5. CONCLUSION

It is concluded that Van Genuchten's closed-form expressions are versatile, convenient and sufficiently accurate for practical purpose. The reliability of these expressions may depend upon physically reasonable estimates of residual water content and saturated volumetric water content, and an adequate soil water retention data base. The unknown parameters in these expressions allow great flexibility in the shape of the hydraulic functions. Therefore when these expressions are determined, the cumbersome handling of unsaturated soil hydraulic properties data in simulation of unsaturated seepage flow problem can be avoided. But, the most important consideration is how to measure these properties accurately in the field or in the laboratory.

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