A Method for Standardizing Parts with Integer Programming

Liu DONG*, Yasuhiro KAJIHARA*, Hirokazu OSAKI* and Yoshiomi MUNESAWA*
(Received October 2, 1995)

A method for standardizing parts is proposed. This method aims to reduce the sort of parts for cutting the manufacturing cost and for improving specifications of parts. Two linear mathematical programming models are proposed for standardizing the parts. One model aims at cutting cost half without causing any degradation of mechanical specifications of parts. The other model aims at doubling mechanical specifications of parts below the acceptable manufacturing cost. These models are formulated in 0-1 integer programming forms. The integer programming model shows which part is common to other parts. An example is shown to demonstrate the use of the developed method.

1. INTRODUCTION

The variety of parts has increased to satisfy varied consumer preference and demand. It has direct influence on the manufacturing cost because it brings the hike of the inventory cost of materials, and forces the kind of assembly JIGs and fixtures to increase. Therefore, the standardization of the parts has become indispensable to reduce the manufacturing cost. So far group technology (GT) [1] and variety reduction (VR) [2] have been used to achieve the above purpose. GT aims at decreasing the kind of parts whose dimension is different. VR aims at decreasing the kind of parts whose specification is different. However, the manufacturing cost has not been considered in these methods. If we can reduce the manufacturing cost of parts more than fifty percent without any degradation of specifications, then the reduction improves our competitive power. If we improve the mechanical specification of parts more than fifty percent under the acceptable manufacturing cost, then the improvement also makes our competitive power stronger. In this paper, we assume that the manufacturing cost is made up of the material cost, processing cost and assembly cost and propose two models of standardizing parts for reducing the manufacturing cost and for improving the mechanical specifications drastically. 0-1 Integer programming models are developed to solve the standardization models.

2. PROPOSED METHOD

In this study, the standardization means to reduce the sort of parts based on the common points of parts. Two methods are proposed. (1) Standardization that aims at reducing the cost and minimizing degradation of mechanical specification. (2) Standardization that aims at manufacturing parts with the maximum specification below the acceptable raised manufacturing cost. The process of the standardization of parts is as follows. First, manufacturing cost and mechanical specifications of all parts are clarified. Next, parts are standardized based on the specifications of each part and the manufacturing cost (material cost, processing cost and assembly cost).

2.1 Symbols and assumptions

The following symbols will be used in this method.

\[ G_i : i-th \text{ product } (i=1, \cdots, N) \]
\[ P_{ij} : j-th \text{ part of product } G_i (j=1, \cdots, NR) \]

* Department of Mechanical Engineering
The following specifications of the parts are considered.

1. Dimensional specifications: length of sides (k=1), the number of hole (k=2), longitudinal and width ratio (k=3), volume (k=4), progression (k=5).

2. The specifications in use: temperature (k=6), speed (k=7), revolutions per minute (k=8), generating power (k=9).

The specifications of each part are measured. K-th specification of the part \( P_k \) is shown by a symbol \( (FX_k) \). If a part \( P_{ij} \) has nothing to do with the k-th specification, we set the \( FX_{ijk} = 0 \).

2.2 Relative specification and manufacturing cost of parts

The dimension and scale of specifications are different. So, we transform each specification into non-dimensional one. The non-dimensional specification is given by the equation (1), and called the relative specification \( f_k \).

\[
\frac{FX_k - \min_{i,s} \{FX_k\}}{\max_{i,s} \{FX_k\} - \min_{i,s} \{FX_k\}} \quad (j=1, \cdots, NR) \quad (k=1, \cdots, NK)
\]  

(1)

The manufacturing cost is made up of material cost, processing cost and assembly cost. In this method, we suppose that following costs are given: material cost \( (CS_{ij}) \), processing cost \( (CM_{ij}) \), assembly cost \( (CA_{ij}) \).

The manufacturing cost of the part \( C_{ij} \) is given in the equation (2) based on the above costs.

\[
C_{ij} = CS_{ij} + CM_{ij} + CA_{ij}
\]  

(2)

Two models are proposed for standardizing the selected parts to reduce the cost. One method aims at cost reduction without causing any degradation of relative specifications. The other method aims at satisfying the upper limit of cost and maximizing the relative specifications.

2.3 Cost reduction amount achieved by the standardization of parts

From above assumption, we evaluate the cost reduction amount that is achieved by the standardization. The reduced amount differs depending on the combination of the parts that are standardized. We use \( D_{ij} \) to show the reduced amount achieved by changing the part \( P_{ij} \) to the part \( P_{iq} \). The reduced amount is given by the equation (3).

\[
D_{ij} = C_{ij} - C_{iq}
\]  

(3)

When the amount \( D_{ij} \) is positive, the manufacturing cost will be cut by the standardization. Each reduced amount of all the combination is shown in Table 1.

2.4 Change of part specification by the standardization

We show all the specifications of all parts in Table 2. The k-th relative specification of part \( P_k \) is shown by \( f_k \). When a part \( P_k \) is changed to a part \( P_{k'} \), change of the relative specification \( DF_{k'} \) is given by the equation (4).

\[
DF_{k'} = \sum_{k=1}^{NK} (f_k - f_k')
\]  

(4)

Change of the relative specification is evaluated by the equation (4) for all the combinations of all parts and shown in Table 2.
Table 3. From the equation (4), when $DF_{ijq}$ is positive, the specification of the part $P_{ij}$ is degraded. Conversely, specification is improved when $DF_{ijq}$ is negative.

2.5 Variables for the standardization

The standardization is carried out in consideration of cost and specifications. We use a variable $X_{ijq}$ for representing whether a part is changed to other one. The variable is defined as follows:

$$X_{ijq} = \begin{cases} 1 : \text{(Part } P_{ij} \text{ is changed to the part } P_{iq}) \\ 0 : \text{(Part } P_{ij} \text{ is not changed to the part } P_{iq}) \end{cases}$$

Variable $X_{ijq}$ is set in all the combinations of all the parts as shown in Table 4.

2.6 Model 1 (The standardization minimizing degradation of specifications)

The aim of the first standardization model is to reduce cost with minimum degradation of relative specifications.

(1) Objective function (minimizing degradation of specifications)

The integer programming model presented below shows which part is common to other parts. The aim of this objective function is to minimize total degradation of specifications. The total degradation of specification ($Z$) is determined by summing up the degradation of specification of all the combinations as shown in Table 3. The minimum value of $Z$ is given in the equation (5).

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{S} DF_{ij} X_{ijq} + DF_{ij} X_{ijq} + \cdots + DF_{ij} X_{ijq}$$

(5)

Constraints for determining the new specifications of all the parts are given by the equation (6) - (8) below.

(2) Constraint 1 (achievement of cost reduction)

The total amount of cost reduction of all the parts should exceed the amount set in the plan model as shown in the equation (6).

$$\sum_{i=1}^{n} \sum_{j=1}^{S} (D_{ij} X_{ijq} + \cdots + D_{ij} X_{ijq}) > T$$

(6)

Here the symbol $T$ represents the target figure of the cost reduction.

(3) Constraint 2

The part ($P_i$) is not able to be made common to more than two kinds of parts. This condition is shown in the equation (7).

$$1 - (X_{ijq} + X_{ijq} + \cdots + X_{ijq}) \geq 0 \quad \text{ (i=1, \ldots, r), \; (j=1, \ldots, S_i)}$$

(7)

(4) Constraint 3

After a standardization, difference ($Z$) of a relative specification of part $P_i$ is given by the next equation.

$$DZ_{ij} = DF_{ij} X_{ijq} + DF_{ij} X_{ijq} + \cdots + DF_{ij} X_{ijq}$$

As the minimum value $LM_i$ which degrades relative specifications of the $P_i$ is given, a restricted condition is given in the equation (8).

$$DZ_{ij} > LM_i \quad \text{ (i=1, \ldots, r), \; (j=1, \ldots, S_i)}$$

(8)
In summary, the entire model minimizing degradation of the specification is presented in a 0–1 integer programming form with all constraints as shown below.

**Objective function:**

\[
\max Z = \sum_{i \in I} \sum_{j \in J} (DF_{i,j} + DF_{i,j+1} + \ldots + DF_{i,m}) 
\]

**Constraints:** equations (6)–(8)

\[
X_{i,j} = 0 \quad \text{or} \quad X_{i,j} = 1, \quad \forall i, j, k
\]

2.7 Model 2 (The standardization maximizing the functional improvement)

For the second model, we set the upper limit of the manufacturing cost and maximize the relative specification of parts without exceeding that limit.

(1) **Objective function (an improvement of a function)**

The sum of improved relative specifications is given by the equation (9).

\[
\min Z = \sum_{i \in I} \sum_{j \in J} (DF_{i,j} + DF_{i,j+1} + \ldots + DF_{i,m}) 
\]

(2) **Constraint 1 (the upper limit of the cost rise)**

We represent the upper limit of raised cost for standardizing parts by a symbol \( T \). The total rise of the manufacturing cost of all parts must be below that upper limit. This relation is shown in the equation (10).

\[
T - \sum_{i \in I} \sum_{j \in J} (DF_{i,j} + \ldots + DF_{i,m}) > 0
\]

(3) **Constraint 2**

Each part is allowed to select one part for the standardization. This relation is shown in the equation (11).

\[
1 - \sum_{i \in I} \sum_{j \in J} (DF_{i,j} + \ldots + DF_{i,m}) = 0 \quad (i = 1, \ldots, r), \quad (j = 1, \ldots, s_i)
\]

The above-mentioned standardization problem is represented by 0–1 integer programming problem. To solve the above problem we use the implicit enumeration, whose algorithm is simple and coding is easy.[3]

3. APPLICATION EXAMPLE

An example is studied to show the effectiveness of the proposed standardization method. The subjects are 16 kinds of personal computers. The relations between the cost and relative specifications of the subjects were checked. We set a target and attempted to minimize the degradation of the relative specifications.

3.1 Product structure trees of computers.

Fig. 1 shows the product structure trees of sixteen computers. Examples of specifications of computers are CPU, memory capacity and so on. As these relative specifications are higher, the cost is higher.

![Product structure tree of a computer](image-url)
The manufacturing cost of main memory and auxiliary memory (HD) is higher than that of the other parts. Main memory has been standardized and only two kinds of memories are used. We attempted to standardize the auxiliary memory. Nine kinds of auxiliary memory (P1, ... , P9) are used.

3.2 Relative specifications

We considered four kinds of specifications, that is, memory capacity, seeking speed, cash memory size and the rotating speed of the auxiliary memory (HD). The relative specifications are given by the equation (1) for all the HDs. Table 5 shows the relative specifications and cost of HDs. From Table 5, the relations between the cost and relative specifications are shown in Figures from 2 to 4 below.

Table 5 Relative specification and cost of auxiliary memories

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>memory capacity</td>
<td>0.062</td>
<td>74800</td>
<td>G6</td>
<td>memory capacity</td>
<td>0.382</td>
<td>75800</td>
</tr>
<tr>
<td></td>
<td>seeking speed</td>
<td>1</td>
<td></td>
<td></td>
<td>seeking speed</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cash memory</td>
<td>0.429</td>
<td></td>
<td></td>
<td>cash memory</td>
<td>0.429</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rotating speed</td>
<td>0.567</td>
<td></td>
<td></td>
<td>rotating speed</td>
<td>0.567</td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>memory capacity</td>
<td>0.152</td>
<td>84800</td>
<td>G7</td>
<td>seeking speed</td>
<td>0.333</td>
<td>99800</td>
</tr>
<tr>
<td></td>
<td>seeking speed</td>
<td>1</td>
<td></td>
<td></td>
<td>cash memory</td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cash memory</td>
<td>0.429</td>
<td></td>
<td></td>
<td>rotating speed</td>
<td>0.329</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rotating speed</td>
<td>0.567</td>
<td></td>
<td></td>
<td>rotating speed</td>
<td>0.329</td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>memory capacity</td>
<td>0.382</td>
<td>118000</td>
<td>G8</td>
<td>memory capacity</td>
<td>0.17</td>
<td>148000</td>
</tr>
<tr>
<td></td>
<td>seeking speed</td>
<td>0.667</td>
<td></td>
<td></td>
<td>seeking speed</td>
<td>0.333</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cash memory</td>
<td>0.074</td>
<td></td>
<td></td>
<td>cash memory</td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rotating speed</td>
<td>0</td>
<td></td>
<td></td>
<td>rotating speed</td>
<td>0.329</td>
<td></td>
</tr>
<tr>
<td>G4</td>
<td>memory capacity</td>
<td>1</td>
<td>198000</td>
<td>G9</td>
<td>memory capacity</td>
<td>0.191</td>
<td>41800</td>
</tr>
<tr>
<td></td>
<td>seeking speed</td>
<td>0</td>
<td></td>
<td></td>
<td>seeking speed</td>
<td>0.667</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cash memory</td>
<td>1</td>
<td></td>
<td></td>
<td>cash memory</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rotating speed</td>
<td>1</td>
<td></td>
<td></td>
<td>rotating speed</td>
<td>0.567</td>
<td></td>
</tr>
<tr>
<td>G5</td>
<td>memory capacity</td>
<td>0.152</td>
<td>53800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>seeking speed</td>
<td>0.667</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>cash memory</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>rotating speed</td>
<td>0.334</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig.2 shows the relation between the cost and the memory capacity. Fig.3 shows the relation between the cost and the seeking speed. Fig.4 shows the relation between the cost and the cash memory size. Fig.5 shows the relation between the cost and the rotating speed of the discs. The total manufacturing cost of HDs were 895 thousand yen. Under these conditions, we set four target figures of cost reduction amount. The first target requires reducing 400 thousand yen in cost. The second 300 thousand yen, the third 200 thousand yen and the fourth 100 thousand yen.
From Table 5, we evaluated the expected reduced cost that is obtained by changing an HD to another type. Expected amount of cost reduction is shown in the Table 6. Next, we evaluated the change of the relative specifications by the equation (4) for all the combinations of the HDs and showed it in Table 7.

3.3 Modeling of the standardization for reducing the manufacturing cost

From Table 7, we set 81 variables of Xijq as shown in Table 8. For example, variable X3 represents a case where the part P1 is changed to the part P2. We considered four target figures of cost reduction amount: 400,000 yen, 300,000 yen, 200,000 yen and 100,000 yen. The entire 0-1 integer programming model for cutting the manufacturing cost more than 400,000 yen is represented as below.

Object: Max $Z = 0X_1 + 9X_2 - 37X_3 + 94X_4 - 91X_5 - 68X_6 - 120X_7 - 109X_8 - 63X_9$

\[ -9X_{10} + 0X_{11} - 46X_{12} + 85X_{13} - 100X_{14} - 77X_{15} - 135X_{16} - 118X_{17} - 72X_{18} \]

\[ + 33X_{73} + 72X_{74} + 26X_{75} + 157X_{76} - 28X_{77} - 5X_{78} - 63X_{79} - 46X_{80} + 0X_{81} \]

Constraints: (1) $0X_1 + 10X_2 + 43X_3 + 123X_4 - 21X_5 + 1X_6 + 25X_7 + 73X_8 - 33X_9$

\[ - 10X_{10} + 0X_{11} + 33X_{12} + 113X_{13} - 31X_{14} - 9X_{15} + 15X_{16} + 63X_{17} - 43X_{18} \]

\[ + 33X_{73} + 43X_{74} + 76X_{75} + 156X_{76} + 12X_{77} + 34X_{78} + 58X_{79} + 106X_{80} + 0X_{81} \geq 400,000$
A Method for Standardizing Parts with Integer Programming

\[
\begin{align*}
(2) \quad & - (X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9) \geq 0 \\
& - (X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18}) \geq 0 \\
& \vdots \\
& - (X_{73} + X_{74} + X_{75} + X_{76} + X_{77} + X_{78} + X_{79} + X_{80} + X_{81}) \geq 0
\end{align*}
\]

Where, subject (1) shows that the reduced cost amount exceeds the target cost reduction amount. Subject (2) shows that one part is not able to be made common to more that two kinds of parts. We did not consider the maximum value \(X_{ij}\) which degrades relative specification of parts.

3.4 The standardization results

We solved 0–1 integer programming problem and showed the result in Table 9. In order to reduce 400,000 yen in cost, it is necessary that \(P_3, P_4, P_5, P_7\) and \(P_8\) be standardized to the part that has a specification of \(P_9\). Then degradation becomes the minimum \((-0.46)\). The relative specification of \(P_4\) recorded a sharp drop. Especially a relative specification of cash memory recorded a sharp drop.

The combination of the standardized parts is shown in Table 9, corresponding to the other target figures of cost reduction amount, 300,000 yen, 200,000 yen and 100,000 yen. Table 6 shows that the relative specification was improved \((+0.46)\) in contrast with the reduced cost when the target figure of cost reduction is 100,000 yen.

<table>
<thead>
<tr>
<th>Amount of cost reduction (×10^2 yen)</th>
<th>Standardization</th>
<th>Change of relative specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>(P_3)→(P_9), (P_4)→(P_9), (P_5)→(P_9) (P_7)→(P_9), (P_8)→(P_9)</td>
<td>-0.46</td>
</tr>
<tr>
<td>300</td>
<td>(P_4)→(P_9), (P_7)→(P_9), (P_8)→(P_9)</td>
<td>-0.48</td>
</tr>
<tr>
<td>200</td>
<td>(P_4)→(P_9), (P_8)→(P_2)</td>
<td>-0.39</td>
</tr>
<tr>
<td>100</td>
<td>(P_8)→(P_9)</td>
<td>+0.46</td>
</tr>
</tbody>
</table>

The total cost before a standardization = 895,000 (yen)

4. CONCLUSION

A part standardization method that considers the specifications of parts and the manufacturing cost is proposed.

(1) The relation between specification and the cost is evaluated.

(2) Parts are standardized based on their specification and manufacturing cost. Two methods are proposed for standardizing the parts. One method aims at cost reduction without causing any degradation of relative specifications of parts. The other method aims at satisfying the upper limit of cost and maximizing the relative specifications.

Reference