

# *Method for Production Planning and Inventory Control in Oil Refinery*

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**Abstract** In this paper, we propose a simultaneous optimization method for inventory control and production planning problem for a chemical batch plant. The plant consists of blending process, intermediate storage tanks and filling process. In the proposed method, the original problem is decomposed into production planning sub-problem and inventory control sub-problem. Then the decision variables are optimized by alternately solving each sub-problem. The solution of the proposed method is compared with that of centralized optimization method. The effectiveness of the proposed method is investigated from numerical computational results.

## 1 Introduction

Recently, high-mix low-volume production has been accelerated through necessity by the diversification of customer's demand. Under these circumstances, having excess stock causes not only the increase in the inventory cost but also decrease in profit because of abrogation of the stock when specification of product is changed. Especially, in lubricant manufacturing factory producing several hundred *kl* or more, proper inventory control is indispensable. Therefore, it is necessary to make production planning that minimizes total cost for productions with minimum inventory considering entire factory at the same time.

Many of past researches about the production management for chemical plants directed to optimize production plan under the conditions of given due date for jobs or amount of production etc [1][2]. However, such the optimization only of production plan is insufficient from the viewpoint of optimization of the entire factory. Therefore, it exists necessity for planning that

considers both the production plan and the inventory control at the same time.

Heretofore, the inventory control and the production planning in the lubricant manufacturing factory have been hierarchically decided [3]. That is, inventory control system that is a superior system outputs the production request to product by which amount of inventory fall below reasonable inventory quantity, and the production planning system decide production plan that is based on the production request. However, such a method can't necessarily optimize total plan.

The simultaneous optimization of inventory control and production planning have been studied [4][5], but these researches are directing to model that is composed only single stage and equipment of given process performance. However, the lubricant manufacturing process is multi stage production composed of the mixture process and the filling process, etc. and intermediate storages between them. And, the processing performance of equipment changes by the production plan of the blending machine and allocating job to the intermediate storages. Therefore, past integrated op-

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timization method can't be applied directly to such a lubricant manufacturing factory.

In this paper, production system considered production process bear peculiar complex restrictive constraints of chemical plant and inventory control is modeled, and optimization method of coordinating inventory control production planning is proposed.

## 2 Optimization problem of production planning and inventory control

### 2.1 Definition of the problem

Problems are for production planning and inventory control of oil refinery plant as shown in Figure 1. The problem treated here is defined in the following.

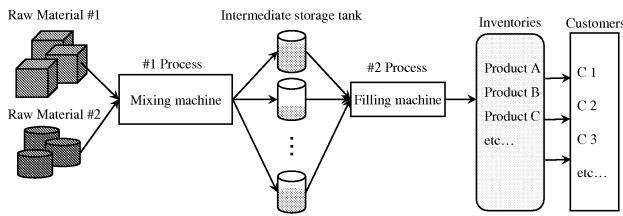


Fig. 1: Chemical Plant

#### 2.1.1 Inventory control

It is assumed that demands from customers are known for planning periods both at present and past time. It is prohibited to have the shortage of inventory. Inventory cost is induced from the amount of product storage. In our research, minimum amount of products storage is assured to have the safety operations. The amount is necessary for safety stock preparing probabilistic changes in demand and occurrence of demands. The amount is calculated based on safety factor relating to service level and past demands data. The service level is a probability to be able to comply with order immediately when there is demand from the customer and the post-process [6]. Cost penalty is added in case of shortage in storage.

#### 2.1.2 Production planning

Production process is composed of blending process, called #1 process, in which raw material group #1 and that of #2 are blended and filling process, called #2 process, in which materials from blending process is packed in predetermined products wares. Both processes require one time period for its productions regardless of its production amounts. The capacity of blending machine is predetermined for one period of time and it is prohibited to blend amounts more than the capacity. On the other hand, filling capacity is also determined beforehand. It is natural that both processes can not process same product at the same time. Between these processes, there installed plural storage tanks acting as buffers storing blended materials in a certain period of time. The capacities for storage tanks are assumed to be sufficiently large. It is prohibited to move blending materials from one tank to another during storing. Materials stored in tanks can be freely diverged into plural jobs in the next filling process. It is natural that the transfer lines from some tank to the following filling process can not be used for other production. In case of change in product kind for storage tank, change over is necessary inducing change over cost. Remaining product, named remaining oil, is stored in the tank for no change over. Filled materials are stored as product stocks.

### 2.2 Mathematical model

When  $t(t = 1, 2, \dots, T)$  is set as a production planning period, the integrated optimization problem of the inventory control problem and the production planning problem can be formulated as the following mixed integer linear programming (MILP) problem.

#### [Notations]

##### Sets:

$\wp$ : Set of products

$\mathfrak{R}$ : Set of tanks

##### Decision variables:

$S_{t,i}^P$ : Inventory level of product  $i$  at the end of time period  $t$  ( $S_{0,i}^P(\forall i)$ : given)

$E_{t,i}$ : Shortage amount of inventory of product  $i$  from amount of safety stock at the end of time period  $t$

$S_{t,k,i}^I$ : Intermediate inventory level of product  $i$  in tank  $k$  at the end of time period  $t$   
 $(S_{0,k,i}^I = 0(\forall k, \forall i))$

$B_{t,k,i}$ : Amount of blending of product  $i$  in tank  $k$  at the time period  $t$

$F_{t,k,i}$ : Amount of filling of product  $i$  in tank  $k$  at the time period  $t$

$$\delta_{t,k,i} = \begin{cases} 1 & (\text{if } B_{t,k,i} > 0) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\gamma_{t,k,i} = \begin{cases} 1 & (\text{if } F_{t,k,i} > 0) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\theta_{t,k,i} = \begin{cases} 1 & (\text{If product } i \text{ is produced using tank } k \\ & \text{at time period } t) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\lambda_{t,k,i} = \begin{cases} 1 & (\text{If intermediate inventory or remaining} \\ & \text{oil of product } i \text{ exist in tank } k \text{ at time} \\ & \text{period } t) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\xi_{t,k,i',i} = \begin{cases} 1 & (\text{If it is switched from } i' \text{ to } i \text{ in tank} \\ & k \text{ at the first of time period } t) \\ 0 & (\text{otherwise}) \end{cases}$$

#### Cost coefficients:

$\mu^P$ : Factor of product inventory cost

$\omega$ : Penalty cost coefficient for shortage amount of inventory from amount of safety stock

$\mu^I$ : Cost coefficient for intermediate inventory

$\phi$ : Cost coefficient for blending set up

$\chi_{i',i}$ : Penalty cost coefficient for product  $i'$  to product  $i$

#### Constant data:

$T$ : Number of time period

$l$ : Maximum lots of blending in one time interval

$F^{max}$ : Maximum amount of filling in one time interval

$D_{t,i}$ : Amount of demand of product  $i$  at period  $t$  (given)

$Q_{t,i}$ : Amount of safety stock of product  $i$  at period  $t$

#### [Problem Description]

$$(P) : \min Z \quad (1)$$

$$Z = \sum_{t,i} \mu^P S_{t,i}^P + \sum_{t,i} \omega E_{t,i} + \sum_{t,k,i} \mu^I S_{t,k,i}^I + \sum_{t,k,i} \phi \delta_{t,k,i} + \sum_{t,k,i',i} \chi_{i',i} \xi_{t,k,i',i} \quad (2)$$

subject to

$$S_{t,i}^P = S_{t-1,i}^P + \sum_k F_{t,k,i} - D_{t,i} \quad (\forall t, \forall i) \quad (3)$$

$$S_{t,i}^P \geq 0 \quad (\forall t, \forall i) \quad (4)$$

$$E_{t,i} \geq Q_{t,i} - S_{t,i}^P \quad (\forall t, \forall i) \quad (5)$$

$$E_{t,i} \geq 0 \quad (\forall t, \forall i) \quad (6)$$

$$S_{t,k,i}^I = S_{t-1,k,i}^I + B_{t,k,i} - F_{t,k,i} \quad (\forall t, \forall k, \forall i) \quad (7)$$

$$\sum_{k,i} \delta_{t,k,i} \leq l \quad (\forall t) \quad (8)$$

$$\sum_{k,i} F_{t,k,i} \leq F^{max} \quad (\forall t) \quad (9)$$

$$\delta_{t,k,i} + \gamma_{t,k,i} \leq 1 \quad (\forall t, \forall k, \forall i) \quad (10)$$

$$\sum_i \theta_{t,k,i} \leq 1 \quad (\forall t, \forall k) \quad (11)$$

$$\sum_i \lambda_{t,k,i} = 1 \quad (\forall t, \forall k) \quad (12)$$

$$\delta_{t,k,i} - \lambda_{t,k,i} \leq 0 \quad (\forall t, \forall k, \forall i) \quad (13)$$

$$\sum_i \delta_{t,k,i} + \lambda_{t-1,k,i} - \lambda_{t,k,i} \geq 0 \quad (\forall t, \forall k, \forall i) \quad (14)$$

$$-\sum_i \delta_{t,k,i} + \lambda_{t-1,k,i} - \lambda_{t,k,i} \leq 0 \quad (\forall t, \forall k, \forall i) \quad (15)$$

$$\lambda_{t-1,k,i'} + \lambda_{t,k,i} - 2 \cdot \xi_{t,k,i',i} \geq 0 \quad (\forall t, \forall k, \forall i', \forall i) \quad (16)$$

$$\lambda_{t-1,k,i'} + \lambda_{t,k,i} - 2 \cdot \xi_{t,k,i',i} \leq 1 \quad (\forall t, \forall k, \forall i', \forall i) \quad (17)$$

$$0 \leq F_{t,k,i} \leq S_{t-1,k,i}^I \quad (\forall t, \forall k, \forall i) \quad (18)$$

$$S_{t,k,i}^I, B_{t,k,i} \geq 0 \quad (19)$$

$$(\forall i \in \varnothing, \forall i' \in \varnothing, \forall k \in \mathfrak{R}, \forall t = 1, \dots, T)$$

Eq.(2) represents objective function, and the first term represents the inventory cost, the second term represents the penalty cost for shortage amount inventory from amount of safety stock, the third term represents the intermediate inventory cost, the fourth term represents blending set up cost and the fifth term represents change over cost. Eq.(3) represents the inventory flow constraint. Eq.(4) represents the amount of the product stock is nonnegative. Eq.(5) and Eq.(6) represent the restrictions concerning the difference between the amount of the safety stock and the amount of the inventory. Eq.(7) represents the intermediate inventory flow constraint. Eq.(8) represents the blending operation capacity constraint. Eq.(9) represents the filling

operation capacity constraint. Eq.(10) represents that the prohibition of the simultaneous processing in blending and filling. Eq.(11) represents that there is one kind of product that can be maintained in each tank at the same time. Eq.(12), Eq.(13), Eq.(14) and Eq.(15) represent the constraint for  $\lambda_{t,k,i}$ . Eq.(16) and Eq.(17) represent the constraint for  $\xi$ . Eq.(18) represents minimum amount and maximum amount of the filling of each product in corresponding tank at each time period. When the safety stock is calculated by using the demand data at  $H$  period of the past, the amount of demand of product  $i$   $D_{t-\tau,i}(\forall t = 1, \dots, T, \forall \tau = 1, \dots, H, \forall i = 1, \dots, I)$  at  $\tau$  period before  $t$  period are given, the amount of safety stock  $Q_{t,i}$  of product  $i$  is computed by using following expressions.

$$\sigma_{t,i} = \sqrt{\frac{1}{H-1} \sum_{\tau=1}^H \left( D_{t-\tau,i} - \left( \frac{\sum_{\tau=1}^H D_{t-\tau,i}}{H} \right) \right)^2} \quad (20)$$

$$Q_{t,i} = m_i \cdot \sqrt{LT_i} \cdot \sigma_{t,i} \quad (21)$$

Here,  $\sigma_{t,i}$  represents the root-mean-square deviation of product  $i$  at the time period  $t$ .  $m_i$  represents safety factor of product  $i$ .  $LT_i$  represents lead time of product  $i$ .

### 2.3 Decomposition of the problem

It is difficult to optimize all variables at the same time. The number of discrete variables may rapidly increase in the model of the inventory control problem and production planning problem for chemical plant. So, in this research, the problem is optimized decomposing original problem to some sub-problems, and applying the decentralized optimization method. Artificial variable  $F_{t,i}^{ICP}$  are introduced to the model from Eq.(1) to Eq.(19), and the constraint Eq.(22) is added. This problem is named problem  $P_2$ .

$$F_{t,i}^{ICP} = \sum_k F_{t,k,i} \quad (\forall t = 1, \dots, T, \forall i \in \varphi) \quad (22)$$

When Eq.(22) is relaxed by using nonnegative Lagrange multiplier  $\nu_{t,i}$ , relaxation problem  $RP_2$  of problem  $P_2$  can be formulated as follows.

$$(RP_2) : \min L \quad (23)$$

$$\begin{aligned} L = & \sum_{t,i} \mu^P S_{t,i}^P + \sum_{t,i} \omega E_{t,i} + \sum_{t,k,i} \mu^I S_{t,k,i}^I \\ & + \sum_{t,k,i} \phi \delta_{t,k,i} + \sum_{t,k,i',i} \chi_{i',i} \xi_{t,k,i',i} \\ & + \sum_{t,i} \nu_{t,i} \left( F_{t,i}^{ICP} - \sum_k F_{t,k,i} \right) \end{aligned} \quad (24)$$

subject to Eq.(4) – (19)

$$\begin{aligned} S_{t,i}^P = & S_{t-1,i}^P + F_{t,i}^{ICP} - D_{t,i} \quad (\forall t, \forall i) \quad (25) \\ (\forall t = & 1, \dots, T, \forall i \in \varphi) \end{aligned}$$

Eq.(25) represents the inventory flow constraint. It is obtained by transforming Eq.(3) using Eq.(22). Lagrangian function  $L$  can be described as follows by consolidating the variable.

$$L = Z_{ICP} + Z_{SP} \quad (26)$$

$$Z_{ICP} = \sum_{t,i} \mu^P S_{t,i}^P + \sum_{t,i} \omega E_{t,i} + \sum_{t,i} \nu_{t,i} F_{t,i}^{ICP} \quad (27)$$

$$\begin{aligned} Z_{SP} = & \sum_{t,k,i} \mu^I S_{t,k,i}^I + \sum_{t,k,i} \phi \delta_{t,k,i} \\ & + \sum_{t,k,i',i} \chi_{i',i} \xi_{t,k,i',i} - \sum_{t,i} \nu_{t,i} \sum_k F_{t,k,i} \end{aligned} \quad (28)$$

When a certain Lagrangian multiplier  $\nu_{t,i}$  are given, the relaxation problem of minimizing Lagrangian function  $L$  can be decomposed to the following sub-problem  $ICP$  and  $SP$ .

$$(ICP) : \min Z_{ICP} \quad (29)$$

subject to Eq.(4) – (6), (25)

$$F_{t,i}^{ICP} \geq 0 \quad (\forall t = 1, \dots, T, \forall i \in \varphi) \quad (30)$$

$$(SP) : \min Z_{SP} \quad (31)$$

subject to Eq.(7) – (19)

Problem  $ICP$  is a sub inventory control problem minimizing weighted sum of inventory cost, penalty cost for shortage amount of inventory from amount of safety stock. Here, artificial variable  $F_{t,i}^{ICP}$  means the amount of the filling of product  $i$  at time period  $t$  that is required of inventory control side from the production side. In the following, the  $F_{t,i}^{ICP}$  is called the amount of the filling demand. Problem  $SP$  is a sub production planning problem minimizing weighted sum of intermediate inventory cost, blending cost and change over cost.

### 3 Decentralized solution algorithm

#### 3.1 Outline of the algorithm

In the algorithm of Lagrangian relaxation, solution process of each sub problem and update of Lagrangian multiplier are carried out alternatively. Basically there is no assurance of convergence of the computation. To prove the problem, the penalty function method by Nishi et al [7] is used. In the method, the distance from the feasible solution is forced to added to the objective function as a penalty cost. As the results, feasibility of the obtained solution can be assured after increasing of penalty weight. The construction of solution process combining *ICP*, inventory control sub-system, and *SP*, production planning sub-system is shown in Figure 2.

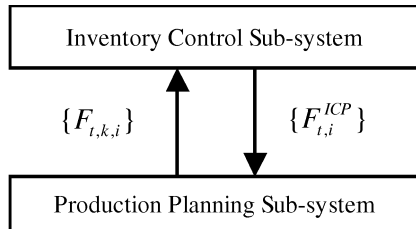


Fig. 2: Structure of optimization system

The algorithm for solving each sub problem is described in the following.

##### Step1 Reading of initial data

Each sub-system retrieves necessary data of the resource constraints, each cost coefficient, and the amount of demand of each period etc. Moreover, the amount of the safety stock of each product for total time horizon is calculated. In addition, the weight  $\rho$  in the penalty term is initialized.

##### Step2 Inventory control planning

The inventory control sub-system decides the inventory control plan to minimize the objective function, and passes amount of the filling demand  $\{F_{t,i}^{ICP}\}$  the production planning sub-system. Here, though it optimizes without considering the production planning sub-system in initial iteration, in iteration since the second times, it receive the amount of filling of each product in corresponding tank at each period  $\{F_{t,k,i}\}$ , it obtained by

optimization of product planning, and decides inventory control plan that minimize weighted sum of the inventory cost, the penalty cost for shortage amount of inventory from amount of safety stock and penalty to gap from feasible solution.

##### Step3 Product planning

The production planning sub-system decides the production plan considering the filling demand  $\{F_{t,i}^{ICP}\}$  to minimize weighted sum of the intermediate inventory cost, the blending cost, the change over cost and penalty to gap from feasible solution. Then, the amount of the filling  $\{F_{t,k,i}\}$  obtained by the optimization of product planning is transferred to the inventory control sub-system.

##### Step4 Evaluation of convergence

If the tentative plan is feasible satisfying Eq.(22), the calculation is ended. Otherwise, algorithm proceeds to Step5.

##### Step5 Update of weight $\rho$

The weight  $\rho$  is increased in  $\Delta\rho$ . Thereafter, algorithm repeats from step 2 to 5 until convergence of evaluation.

First of all, each sub-system retrieves data as a preparation. Afterwards, the inventory control sub-system and the production planning system optimize the original problem by repeating optimization of the each problem according to each objective function. Here, each sub-system exchanges the amount of filling of corresponding product at each period to satisfy the consistency of Inventory control plan and production plan. Each sub-system adds penalty to the difference between filling plan preferable for each sub-system and the filling plan obtained from another sub-system to each objective function. The solution of original problem is gradually approaches to feasible solution by increasing the value of the penalty coefficient gradually fill the iteration ends. The flow of the algorithm is shown in Figure 3.

In the following, optimization of each sub-system will be stated.

### 3.2 Inventory control sub-system

#### 3.2.1 Inventory control sub-system

In the target chemical plant, due to the restrictions for usable number of tanks and maximum number of

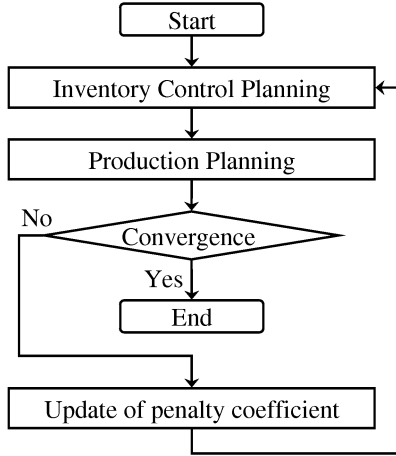


Fig. 3: Flow chart of proposed method

lots in one time interval are predetermined. The feasibility of inventory plan determined by inventory control plan is affected from demand of filling operations for each time period. To reflect the effect, the difference between filling plan by inventory and that by production plan is added to the objective function of inventory planning as the penalty factor. Optimization problem *ICP* in the inventory control sub-system can be formulated by adding a binary variable  $\eta_{t,i}$  as the following mixed integer linear programming problems.

$$\eta_{t,i} = \begin{cases} 1 & (\text{if } F_{t,i}^{ICP} > 0) \\ 0 & (\text{otherwise}) \end{cases} \quad (\forall t, \forall i)$$

$$(ICP) : \min Z_{ICP} \quad (32)$$

$$Z_{ICP} = \sum_{t,i} \mu^P S_{t,i}^P + \sum_{t,i} \omega E_{t,i} + \sum_{t,i} \rho |\eta_{t,i} - \Gamma_{t,i}| \quad (33)$$

subject to Eq.(4) – (6), (25), (30)

The third term of right side in equation (33) is the artificially added factor representing difference between the value of filling plan  $\eta_{t,i}$  by *ICP* and  $\Gamma_{t,i}$  that by *SP*.  $\Gamma_{t,i}$  represents the presence of the filling plan of each product at each period in production plan computed by production planning sub-system. If  $\sum_k \gamma_{t,k,i} \geq 1$ , then  $\Gamma_{t,i} = 1$ , otherwise zero.

Increasing the value of weight  $\rho$  after solving *SP* sub problem, it becomes possible to derive feasible solution. However, if we use only the penalty method, convergent time apt to be large due to the other constraints

for the plan. To overcome the difficulties modified constraint for *SP* sub-system is added to *ICP* sub-system as follows.

$$F_{1,t}^{ICP} = 0 \quad (\forall t = 1, \dots, T) \quad (34)$$

$$\sum_i F_{t,i}^{ICP} \leq F^{\max} \quad (\forall t = 1, \dots, T) \quad (35)$$

$$\sum_i \eta_{t,i} \leq K \quad (\forall t = 1, \dots, T) \quad (36)$$

Eq.(34) represents constraint of the amount of the filling in at the first time period, and it is obtained from the initial condition of  $S_{t,k,i}^I$ , Eq.(18), and Eq.(22). Eq.(35) represents the upper bound of the amount of the filling, and it is obtained from Eq.(9) and (22). Eq.(36) represents upper bounds of number of product kind that can be the filling processing for one time period. This constraint can be obtained from problem setting of only one kind of product can be processed at the same time in each tank. The optimal solution can be obtained by using a commercial solver because *ICP* is mixed integer linear programming problem including continuous variable.

### 3.3 Production planning

In the target chemical plant, capacity of production is affected by production plan because intermediate storage of materials in tanks and divergence of jobs in the filling process may be occurred. The examples are shown in Figure 4 and Figure 5, where only one tank is usable for intermediate storage.

Period	1	2	3	4
Tank	Blend {A}	Fill {A}	Fill {A}	Fill {A}

Fig. 4: Change of the producing capacity

Fig.4 represents a Gantt chart of production plan that designed filling of product A from second terms to fourth term. On the other hand, Fig.5 represents one that designed filling of product A from second terms to third term and it of product B at fourth term.

In both cases, one lot filling is designed from second term to fourth term. However, feasible solution is obtained in Fig.4 and infeasible one is obtained in Fig.5 due to the prohibition by constraints. Thus, the

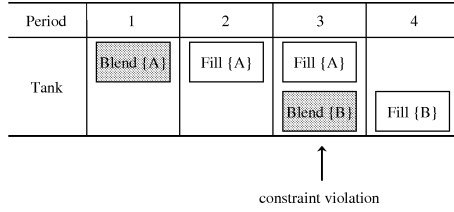


Fig. 5: Change of the producing capacity

capacity of each time interval in production varies according to production plan. In the planning for inventory control, it is impossible to reflect such change in production capability. As the result, calculated filling request made by inventory control planning may be infeasible for production. So it is necessary to revise the calculated filling request from inventory for the total feasibility of obtained results by updating penalty factor  $\rho$ .

In the proposed method, feasible solution is created by  $F_{t,i}^{ICP}$  is deemed to filling job of product kind  $u_i (= i)$ , due date  $d_t (= t)$ , amount of filling  $g_j (= F_{t,i}^{ICP})$ , and the deviation from due date is allowed with penalty cost is named deviation from due date penalty cost. The production plan sub-system can make feasible production plan considering the given filling demand, by using this method. The reason to give production planning sub-system the deviation from due date penalty cost is the production capacity changes greatly by changing the processing time period of blending lots and filling lots because these processes are batch process. The following notations are introduced into problem  $SP$ .

#### Sets:

$J$ : Set of filling jobs.

$\pi_i$ : Set of filling jobs that satisfy  $u_j = i$

#### Decision variables:

$h_{j,t,k}$ : Amount of filling job  $j$  in tank  $k$  at time period  $t$

$$x_{j,t,k} = \begin{cases} 1 & (\text{if } h_{j,t,k} > 0) \\ 0 & (\text{otherwise}) \end{cases}$$

$a_j$ : Filling date of filling job  $j$

$r_j$ : Amount of filling of product  $i$  in tank  $k$  at the time period  $t$

Then, the following constraints are added to problem  $SP$ .

$$\sum_{t,k} h_{j,t,k} = g_j \quad (\forall j \in J) \quad (37)$$

$$\sum_{t,k} x_{j,t,k} = 1 \quad (\forall j \in J) \quad (38)$$

$$a_j = \sum_{t,k} t \cdot x_{j,t,k} \quad (\forall j \in J) \quad (39)$$

$$r_j = |a_j - d_j| \quad (\forall j \in J) \quad (40)$$

Eq.(37) and Eq.(38) represent that all the filling jobs are processed. Eq.(39) represents the definition constraint of  $a_j$ . Eq.(40) represents the constraint for  $r_j$ . Moreover, the existing constraint is converted as follows.

$$S_{t,k,i}^I = S_{t-1,k,i}^I + B_{t,k,i} - \sum_{j \in \pi_i} h_{j,t,k} \quad (\forall t, \forall k, \forall i) \quad (41)$$

$$\sum_{j,k} h_{j,t,k} \leq F^{\max} \quad (\forall t) \quad (42)$$

$$\delta_{t,k,i} + x_{j,t,k} \leq 1 \quad (\forall t, \forall k, \forall i, \forall j \in \pi_i) \quad (43)$$

$$0 \leq \sum_{j \in \pi_i} h_{j,t,k} \leq S_{t,k,i}^I \quad (\forall t, \forall k, \forall i) \quad (44)$$

$$(\forall i \in \emptyset, \forall k \in \mathfrak{R}, \forall t = 1, \dots, T)$$

Eq.(41) represents inventory flow constraints. It is obtained by transforming Eq.(7). Eq.(42) represents the filling operation capacity constraint. It is obtained by transforming Eq.(9). Eq.(43) represent that the prohibition of the simultaneous processing in blending and filling. It is obtained by transforming Eq.(10). Eq.(44) represents minimum amount and maximum amount of filling of each product in corresponding tank at each time period. It is obtained by transforming Eq.(18).

When it occurs the deviation from due date inventory planning sub-system can't create feasible solution that satisfy the filling plan is obtained by optimization of product planning because of out of inventory. It causes delay of convergence of solution. So the following constraint that represents the lowest amount to be filled before each period to out of inventory is not caused is added to problem  $SP$ . This constraint is obtained from Eq.(3) and Eq.(4)

$$S_{0,i}^P + \sum_{t'=1}^t \sum_k \sum_{j \in \pi_i} h_{j,t',k} - \sum_{t'=1}^t D_{t',i} \geq 0 \quad (\forall t, \forall i) \quad (45)$$

$$(\forall i \in \emptyset, \forall t = 1, \dots, T)$$

Therefore, problem  $SP$  can be formulated as a problem to minimize weighted sum of the intermediate inventory cost, the blending set up cost, the changeover

cost, and the deviation from due date penalty cost to the filling demand as follows. Here,  $\kappa_1$ ,  $\kappa_2$  in fourth term of Eq.(47) are added artificially to match the value of penalty that is added to problem  $SP$  to the value of penalty that is added to problem  $ICP$  because those penalties are different.

$$(SP) : \min Z_{SP} \quad (46)$$

$$Z_{SP} = \sum_{t,k,i} \mu^I S_{t,k,i}^I + \sum_{t,k,i} \phi \delta_{t,k,i} + \sum_{t,k,i',i} W_{i',i} \xi_{t,k,i',i} + (\kappa_1 \cdot \rho + \kappa_2) \sum_j r_j \quad (47)$$

subject to Eq.(8), (11) – (17), (19), (37) – (45)

It is difficult to attain strict optimization because of the objective function of problem  $SP$  contains the changeover cost that depends on order of operation. Then, the production plan is optimized by using the algorithm of the following SA (Simulated annealing method)[8] is the following algorithms. First of all, to expand the search space of the solution, the constraints of Eq.(8) and Eq.(11) are relaxed and added to the objective function as a penalty like in Eq.(49).

$$(SP) : \min Z_{SP} \quad (48)$$

$$Z_{SP} = \sum_{t,k,i} \mu^I S_{t,k,i}^I + \sum_{t,k,i} \phi \delta_{t,k,i} + \sum_{t,k,i',i} W_{i',i} \xi_{t,k,i',i} + (\kappa_1 \cdot \rho + \kappa_2) \sum_j r_j + \sum_t \zeta \nu_t + \sum_{t,k} \epsilon \alpha_{t,k} \quad (49)$$

subject to Eq.(12) – (17), (19), (37) – (45)

$$\nu_t \geq \sum_{k,i} \delta_{t,k,i} - l \quad (\forall t, \forall k, \forall i) \quad (50)$$

$$\nu_t \geq 0 \quad (\forall t, \forall k, \forall i) \quad (51)$$

$$\alpha_{t,k} \geq \sum_i \theta_{t,k,i} - 1 \quad (\forall t, \forall k, \forall i) \quad (52)$$

$$\alpha_{t,k} \geq 0 \quad (\forall t, \forall k, \forall i) \quad (53)$$

$$(\forall i \in \varnothing, \forall k \in \mathfrak{R}, \forall t = 1, \dots, T)$$

Here,  $\zeta$  represents the penalty cost for the violation of blending operation capacity constraint.  $\epsilon$  represents the penalty cost for the violation of resource constraint about tanks.

### Step1 Initial allocation of the filling jobs

To satisfy due date, the filling jobs are allocated to a tank.

### Step2 Production planning

Problem  $SP$  is solved by using a commercial solver, and the production plan that is satisfy allocations of the jobs that is decide in the previous step is obtained.

### Step3 Evaluation of production plan and adoption judgment

The production plan is evaluated by using Eq.(49). And, the adoption judgment of the production plan is decided according to the rule of the SA method.

### Step4 Neighborhood operation

To satisfy the constraint of Eq.(45), the allocation of a filling job that is to select at random is changed at random. And a regulated frequency repeats from step 2 to step 4.

## 4 Numerical experiments

### 4.1 Centralized method

To check the validity of the proposed method, results are compared with the centralized total optimization method. The compared method is based on SA method. In the centralized method, once filling plan, the time of filling of each product kind and tank number are made and then the volume of filling is determined by SA algorithm. And at the same time blending volumes are also determined. The procedure of the centralized method is given as follows.

### Step1 Initial allocation of $\gamma_{t,k,i}$

All  $\gamma_{t,k,i}$  that represent the presence of the filling plan of each product in tank at each period are decided. Here, those are decided as the tamp ahead plan and the constraints from Eq.(3) to Eq.(19) and  $\gamma_{t,k,i} = 1$  are satisfied.

### Step2 Decision of inventory plan and production plan

The inventory control plan and the production plan that minimize Eq.(2) and satisfy  $\gamma_{t,k,i}$  are decided in the previous step using a commercial solver.

### Step3 Evaluation of production plan and adoption judgment

The inventory control plan and the production plan obtained in Step3 are evaluated. And, the



adoption judgment of the production plan is decided according to the rule of the SA method.

**Step4 Neighborhood operation**

$t, k,$  and  $i$  are selected at random, and  $\gamma_{t,k,i}$  is reversed.

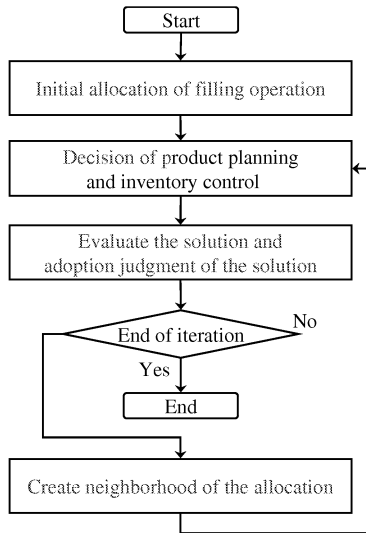


Fig. 6: Flow chart of centralized method

**4.2 Numerical Examples**

Numerical experiments are conducted for 3 cases examples shown in Table 1. The demand of each product at each period in the planning term and H term of past immediately before the planning term is generated using normal random number based on root-mean-square deviation and average amount of demand are shown in Table 2. The changeover cost is shown in Table 3. The product data is shown in Table 4. Other data is shown in Table 5.

Table. 1: Examples

example	Number of Time Period	Number of Product	Number of Tank
CASE1	5	8	6
CASE2	5	9	7
CASE3	5	10	8

Table. 2: Parameters for the making of demand

product	root-mean -square deviation	average amount of demand
A	3.8	6.2
B	4.0	6.0
C	2.5	2.3
D	2.7	4.3
E	12.4	17.4
F	16.9	32.2
G	4.8	10.8
H	7.0	5.0
I	8.0	7.0
J	6.0	8.0

Table. 3: Changeover cost data

from\to	1	2	3	4	5	6	7	8	9	10
1	0	30	40	50	60	50	70	30	60	80
2	30	0	50	60	70	70	80	60	50	60
3	40	50	0	70	80	10	20	30	70	40
4	50	60	70	0	90	100	40	90	20	50
5	60	70	80	90	0	30	70	70	30	60
6	50	70	10	100	30	0	50	80	60	30
7	70	80	20	40	70	50	0	20	70	50
8	30	60	30	90	70	80	20	0	50	40
9	50	40	30	70	50	60	70	20	0	60
10	60	70	50	40	30	20	80	60	30	0

Table. 4: Product Data

product	$m_i$	$LT_i$	$S_{0,i}^P$
A	4	7	21
B	2	7	24
C	3	7	15
D	7	7	15
E	6	7	40
F	4	7	80
G	5	7	30
H	4	7	30
I	5	7	50
J	6	7	60

**4.3 Experimental result and consideration**

The system developed uses CPLEX8.0 as a commercial solver. An initial value of the penalty coefficient  $\rho$

**Table. 5: Other data**

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$H: 5, \mu^I: 1, \mu^P: 4, l: 4$   
 $F^{\max}: 500, \omega: 3, \phi: 100, \kappa_1: \frac{1}{3}$   
 $\kappa_2: 10, \zeta: 1000, \epsilon: 3000$

---

is set to be zero and  $\Delta\rho = 300$ . Moreover, the parameter of the SA method that is used when the system solve the problem  $SP$  in the proposed method and the centralized method is shown in Table 6.

Table. 6: Parameters for the SA

parameter	Proposed Method( $SP$ )	Centraized Method
Maximum Temperature	3000	10000
Minmum Temperture	30	100
Cooling Period	20	50
Cooling Rate	0.9	0.9

The comparing of computation time and evaluation value of plans that are optimized by the centralized method and the proposed method are shown in Figure 7 and Figure 8. Moreover, Gantt charts that are obtained by those methods are shown in Figure 9 and Figure 10. Here, the alphabets show the kind of product and numbers show the value of the processing in these figures. It can be confirmed that the proposed method obtains the better solution in a short computation time compared with centralizd method. This reason is thought that the SA method used for the centralized method that can obtain the optimal solution in infinite time is can't optimize problem enough in limited time because it is a method of searching for the solution at random. On the other hand, it is thought that the better solution in short time can be obtained by the proposed method because it is possible to search for solutions near optical solution by iterating the optimization of the each sub-problem and the information exchange between sub-systems. The effectiveness of the proposed method to such problems is investigated by these results.

### 4.4 Conclusion

In this paper, decomposed solution method is proposed to solve the chemical plant with two processes and intermediate storage between them. In the method,

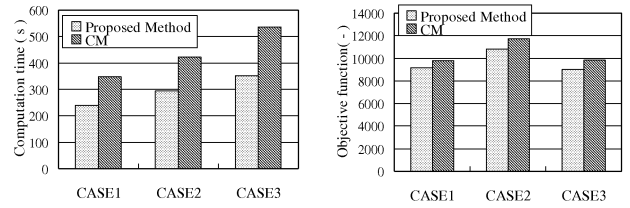


Fig. 7: Comparison of computation time

Fig. 8: Comparison of evaluation value

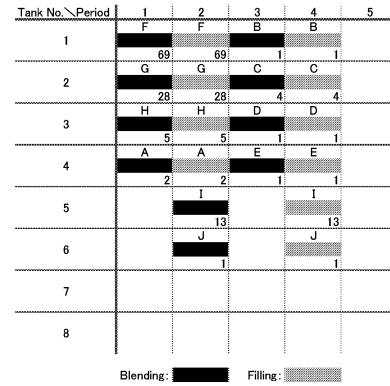


Fig. 9: Ganttchart(Concentrated method)

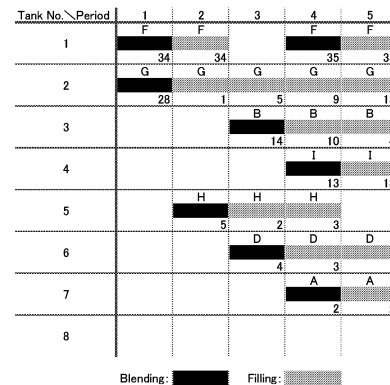


Fig. 10: Ganttchart(Proposed method)

inventory control planning and production planning are made alternatively. The proposed method is revealed to show the better solution in a short computation time compared with centralized method. The extension of the proposed method to improve solution optimality and reflection of procurement which leads to the total supply chain solver.

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