Journal of the Faculty of Environmental Science and Technology, Okayama University
Vol. 1, No. 1, pp.47-53, March 1996

Computational water analysis in an artificial lake: Kojima Lake case

Toru Sasaki, Hirofumi Ishikawa
Tsuyoshi Kajiwara and Masaji Watanabe *

(Received November 17 , 1995)

Abstract

We treat the problem of water pollution by the method of a mathematical model. We illustrate the method of analysis with Kojima Lake. We analyze in-flow and out-flow of the lake, compute numerical solutions of the governing equations of the water flow and the pollutant. The simulation leads to the conclusion concerning the figure of Kojima Lake.

KEYWORDS: Kojima lake, Water analysis, Finite element method

1. Introduction

In consequence of intersection between human beings and their surroundings, the contamination of rivers, lakes, and sea with waste matters and chemical materials has increased during that past quarter century. Especially, the overnourishment leads to surplus breeding in harmful planktons against aquatic life.

In this article, we have treated the problem of water pollution by the method of a mathematical model formed as a system of partial differential equations, which will be made use of controlling the environment; the partial differential equations have numerically been solved by finite element method. We have studied the case of Kojima lake in this article. Kojima lake, located in Okayama prefecture and fronting on Kojima bay in the Inland Sea, was constructed in 1959, for the purpose of ensuring irrigation water; it is an artificial and freshwater lake. It has a water lock (200 meters in length), which prevents sea water from flowing back, so that the draining off of the contamination is considerably obstructed by the lock. This article is composed of the analysis of the amount of inflowing water from the rivers into Kojima lake, the simulation of the flow of water in the lake, and that of convection diffusion of contamination there.

2. Analysis of The Amount of Inflowing Water

In order to study the hydrologic cycle of Kojima lake, we have investigated the amount of flowing water for the rivers which are inflowing in Kojima lake. In this article, as we treat the stationary state case, we can, therefore, assume that the amount of outflowing water through the lock is as much as the totally amount of inflowing water. It is important to investigate those amounts since the boundary conditions of our differential equations are principally determined by them.

When a river basin is determined, the amount of flowing water for the river can be estimated by the quantity of precipitation on its basin. By using topographical maps on a scale of 1 to 50,000, all the basins

*Department of Environmental and Mathematical Sciences, Faculty of Environmental Science and Technology, Okayama University, Okayama, 700 Japan.

the river of which flows into Kojima lake have been described; for the cases of Sasagase river and Kurashiki river, those are shown in Figure 1 and Figure 2.

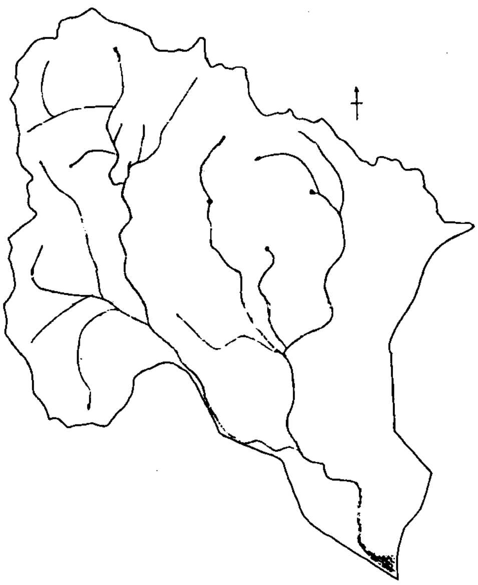

Figure 1
The outline map of the Sasagase basin

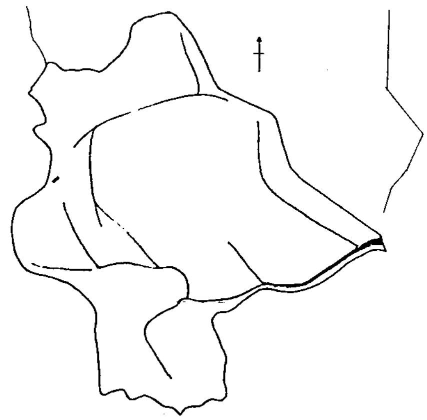

Figure2
The outline map of the Kurashiki basin

In this article, we have studied the cases of three kinds of the amount of flowing water:

(i) the mean amount a day, which is averaged through a year

(ii) the maximal amount a day, which is averaged through the month on a rainy season that has the maximal rainfall in a year

(iii) the minimal amount a day on a period of water shortage

We use the rainfall data at Okayama, Takahashi, Fukuwatari, Kurashiki, and Tamano observation points for past six years (1989-1994) offered by the Okayama Meteorological Observatory. The estimation of the quantity of precipitation on the Sasagase basin, and the Kurashiki basin depended on the mean of five rainfall data at Okayama, Takahashi, Fukuwatari, and Kurashiki points, and on the mean of two data at Okayama, and Kurashiki points, respectively. For each river, the mean amount (i) is calculated by the product of the area of its basin and the effective rainfall after subtracting the amount of evapotranspiration, assuming that 650 mm/km^2 a year from the annual rainfall. The maximal amount (ii) is also calculated as the same way of (i). The minimal amount (iii) is estimated at the product of the area of its basin and the quantity of grand water, assuming that 1 mm/km^2 a day . We have tabulated above three amounts (i), (ii), and (iii) for Sasagase, Kurashiki, and Kamo rivers in Table 1.

Table 1.
The amount of flowing water and the area of river basin

Rivers		Sasagase	Kurashiki	Kamo
Area of basin	km^2	290	150	60
Mean amount	$10^5 m^3/day$	4.8	2.4	0.9
Maximal amount	$10^5 m^3/day$	20.6	10.5	3.9
Minimal amount	$10^5 m^3/day$	2.9	1.5	0.6

3. Analysis of The Flow of Water

In this section, we analyze the flow of water in Kojima lake the outline of which is shown in Figure 3 by using Navier-Stokes equation. As for inflowing water from the estuaries, we have adopted the data in section 2, and the boundary conditions on the estuaries and the lock of the bank separating fresh water and salt one are determined by them.

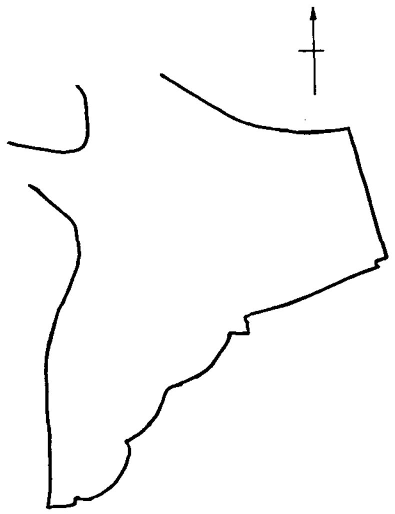

Figure 3.
Kojima lake

The mean depth of Kojima lake measures up to 1.6 m and is very shallow. Since the depth of water does not vary much, we may use 2-dimensional equation for our analysis. Navier-Stokes equation is non-linear and is hard to deal with. The water in Kojima lake flows so slowly that the non-linear term can be neglected in this equation. Moreover, since the influence of vortexes in the lake can be practically disregarded, we may assume that there exists a scalar function Φ such that the vector field $(u(x,y), v(x,y))$ describing the water flow equals to $-\operatorname{grad}\Phi$. In this case, the equation becomes to the following Laplace's equation:

$$\Delta\Phi = 0$$

We also set the constant velocity boundary condition on the estuaries and the bank. For the convenience of calculation, we simplify the shape of Kojima lake as in Figure 4; it is divided into the meshes also shown in Figure 4.

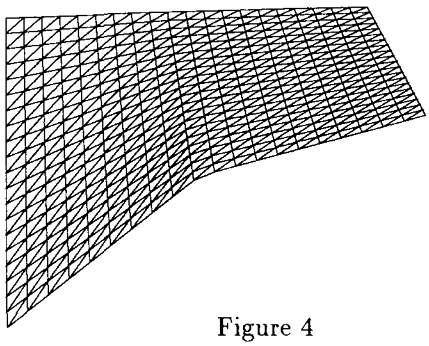

Figure 4
Meshes

In Figure 4, (1), (2), or (3) points out the estuary of Sasagase river, the estuary of Kurashiki river, or the new lock on the bank; the width of above two estuaries or the gate measures up to $l_1 = 1,110m$, $l_2 = 540m$, or $l_3 = 200m$, respectively.

Let $\mathbf{n}$ be a normal vector at a point of the boundary directed to the inner part of the area. We set the boundary condition as the following form:

$$\frac{\partial\Phi}{\partial\mathbf{n}} = f$$

The velocity function f is decided as follows. $f = -r_1$ on the estuary of Kurashiki river, $f = -r_2$ on the estuary of Sasagase river and $f = r_3$ on the lock (3), where r_1 and r_2 stand for the water velocity at the esturies of both rivers, and r_3 is determined by the equation $r_3 = \dfrac{r_1 l_1 + r_2 l_2}{l_3}$. In other places on the boundary, we put $f = 0$. The function f on the boundary satisfies the following identity, which is necessary for the Neumann type boundary condition.

$$\int_\Omega f(s)\,ds = 0$$

Here we use the symbol Ω for Kojima lake modified in Figure 4 when we treat the lake in mathematical sense. $\partial\Omega$ stands for the boundary of Ω.

We use the finite element method for the computational analysis of this problem. This equation is transformed into the weak form by multiplying a piecewise differentiable function φ and integrating over Ω.

$$\int\int_\Omega \left(\frac{\partial\varphi}{\partial x}\frac{\partial\Phi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\Phi}{\partial y}\right) dxdy = -\int_{\partial\Omega} \varphi f\,ds$$

Let $\varphi_{i,j}$ be the piecewise linear function which takes a value 1, or 0 at (i,j)'s lattice point, or the others lattice ones, respectively. Let $\{c_{k,l}\}_{i,j}$ be a set of real constants. We substitute $\varphi_{i,j}$ to the above φ, $\sum_{k,l} c_{k,l}\varphi_{k,l}$ to Φ.

Then we get the following simultaneous linear equation with respect to $\{c_{k,l}\}$'s.

$$\sum_{k,l}\left(\int\int_\Omega \left(\frac{\partial\phi_{i,j}}{\partial x}\frac{\partial\phi_{k,l}}{\partial y} + \frac{\partial\phi_{i,j}}{\partial x}\frac{\partial\phi_{k,l}}{\partial y}\right) dxdy\right) c_{k,l} = -\int_{\partial\Omega} \phi_{i,j} f\,ds$$

This can be solved by the sweeping method. We omit the details.

Differentiating the approximate solution $\Phi = \sum_{i,j} \Phi_{i,j}$, we get the vector field of the flow. This vector field by the lines is displayed in Figure 5.

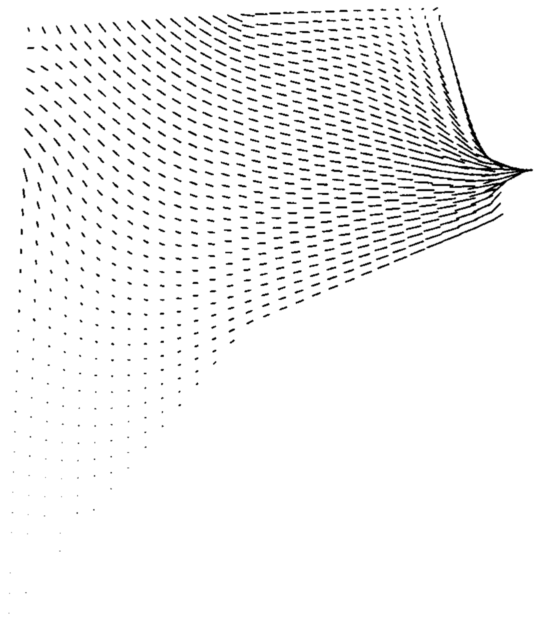

Figure 5.
The flow of water in Kojima lake

4. Convection Diffusion analysis of Contamination

We assume that the contamination flows into Kojima lake from Sasagase river and Kurashiki river. We study the diffusion of the pollutant in Kojima lake by convection-diffusion equation. In this article, we treat only the stationary solutions.

We denote by $c(x, y)$, and $(u(x, y), v(x, y))$ the density of the pollutant at (x, y), and the velocity vector of flow at (x, y), respectively. The velocity (u, v) has been simulated in section 3. We also denote by κ the diffusion constant of contamination. Then the function c of (x, y) satisfies the following equation:

$$\kappa \Delta c = u c_x + v c_y$$

We set the boundary condition as follows:

$$\begin{aligned} c &= \text{constant}, && \text{on the estuaries} \\ \frac{\partial c}{\partial \mathbf{n}} &= -r_0 c, && \text{on the lock (Robin condition)} \\ \frac{\partial c}{\partial \mathbf{n}} &= 0, && \text{otherwise} \end{aligned}$$

We determine the value of the diffusion constant in a standpoint of chemistry and hydrodynamics. We transform the above equation into weak form as in section 3.

$$\int\int_{\Omega}\left(\frac{\partial\varphi}{\partial x}\frac{\partial c}{\partial x}+\frac{\partial\varphi}{\partial y}\frac{\partial\varphi}{\partial y}\right)dxdy+\frac{1}{\kappa}\int\int_{\Omega}\varphi(uc_x+vc_y)\,dxdy=\int_{\partial\Omega}\varphi\frac{\partial c}{\partial\mathbf{n}}\,ds$$

We substitute $\varphi_{i,j}$ to φ, $\sum_{k,l}c_{k,l}\varphi_{k,l}$ to c. By $\Phi=\sum_{\mu\nu}K_{\mu,\nu}\varphi_{\mu,\nu}$, we get $u=\sum_{\mu,\nu}K_{\mu,\nu}\frac{\partial\varphi_{\mu,\nu}}{\partial x}$ and $v=\sum_{\mu,\nu}K_{\mu,\nu}\frac{\partial\varphi_{\mu,\nu}}{\partial y}$. We substitute these to u and v. Then we have the following simultaneous equation with respect to $\{c_{k,l}\}$.

$$\begin{aligned}&\sum_{k,l}\left(\int\int_{\Omega}\left(\frac{\partial\varphi_{ij}}{\partial x}\frac{\partial\varphi_{kl}}{\partial x}+\frac{\partial\varphi_{ij}}{\partial y}\frac{\partial\varphi_{kl}}{\partial y}\right)dxdy\right)c_{kl}\\+\;&\frac{1}{\kappa}\sum_{k,l}\left[\sum_{\mu,\nu}K_{\mu,\nu}\int\int_{\Omega}\varphi_{ij}\left(\frac{\partial\varphi_{\mu,\nu}}{\partial x}\frac{\partial\varphi_{kl}}{\partial x}+\frac{\partial\varphi_{\mu,\nu}}{\partial y}\frac{\partial\varphi_{kl}}{\partial y}\right)dxdy\right]c_{kl}\\=\;&\int_{\partial\Omega}\varphi_{i,j}g\,ds\end{aligned}$$

Solving this equation numerically, we get $\{c_{i,j}\}$. The graphical expression of the simulation carried out in the case of mean amount of flowing water (i) is given in Figure 6; these graphs are projected from two distinct points of view.

The simulation quantitatively shows that the pollutant tends to accumulate in the southwest part of Kojima lake.

In order to obtain better results, we should study the non-stationary models in future.

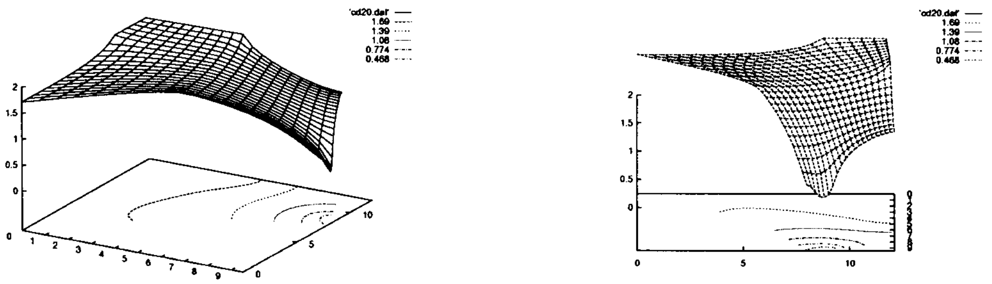

Figure 6
The numerical solution of the convection-diffusion equation

Acknowledgment

The authors wish to express their gratitude to Professor A. Nagai for his fruitful lecture on hydrologic cycle, and to Professor T. Iwachido for his helpful comment on diffusion coefficients. They also thank the Okayama Meteorological Observatory for her offer of weather data.

References

[IUK] Imai, T., T. Umetsu and M. Kawahara; A predictive control method for water-pollution problems using finite element method, Computer Modeling in Ocean, Engineering 91, Arcilla et.al., Balkema, (1991)

[K] Kosaka, K. (edit); Environments of Setonaikai Sea, Koseisha Koseikaku (1985) (in Japanese)

[N] Nagai, A.; Analysis of outflowing in mountainous river basins with long terms and the standard decreasing curve for the dams of Setouchi area, The study of the standardization in the ratio of droughty water discharge, Report of a Grant-in-Aid for Science Research from the Ministry of Education of Japan, edited by N. Tanakamaru, 73-100, (1993) (in Japanese)