Ultrasonic Inversion for Determining Crack Size in a Solid

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A linear scattering inverse method based on the Kirchhoff approximation is formulated to determine the location and size of a crack in a solid. A characteristic function, which defines the size of a crack, can be reconstructed from the inverse Fourier transform of scattered amplitudes at far field. The inverse method is applied to ultrasonic data scattered by a crack in an aluminum specimen. Agreement between reconstructed characteristic functions and exact ones is not good enough, because experimental conditions do not coincide with theoretical ones. We can, however, evaluate the location and size of a crack from sharp minimum points reproduced at crack tips.

1. INTRODUCTION

Composite structures, which are widely used in many significant applications in industry, consist of different elastic materials formed in layers or inclusions. Two different solids are bonded together on the common interfaces, but it often happens that the bonding is imperfect, and cracks or delaminations occur at the interface. The presence of these defects makes materials vulnerable to failure due to propagation or growth of defects. A quantitative nondestructive detection of interface defects is of great importance in predicting useful life and failure properties of composite materials. One of the most promising methods for the detection utilizes the scattering of elastic waves, which could convey quantitative information on the defects. The final goal of our research will be a quantitative nondestructive evaluation of cracks in composite materials. In this paper, however, we take the first step by considering the inversion problem on determining crack size in a homogeneous elastic solid.

Ultrasonic methods have successfully been applied to determine crack size in a solid. The first arriving waves at the receiving transducer are due to the diffracted waves at the crack tips. The interference of these waves exhibits a periodic modulation in the frequency domain, which relates to the crack size. The periodicity of the scattered spectrum has been used to characterize a crack-like planar defect [1], a circular crack [2], an elliptic crack [3] and an interface crack [4]. The principle of estimation of crack size using diffracted waves is very simple and accurate. The method can not, however, be applied to multiple cracks, for which the interference of diffracted waves is too complicated to perform the inversion. This paper presents a scattering inverse method, which can also be applicable to multiple cracks. The method is based on the inverse Kirchhoff approximation [5]. A scattered far field has linear relation with the Fourier transform of the characteristic function, which represents crack distributions. The size or distribution of cracks can be obtained by means of the inverse Fourier transform of the scattered far field. The proposed inversion method is applied to ultrasonic waves measured in a water immersion test.

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2. INVERSE SCATTERING THEORY

2.1 The inverse problem

We consider a crack $S$ subjected to a plane incident wave $u^{in}$ in a 2-D isotropic homogeneous elastic solid $D$ of infinite extent as shown in Fig. 1. The incident wave is a time-harmonic plane $S$ wave given by

$$u^{in}(x) = A^{in} d_{in}^j p_{in}^j x_j$$

where $A^{in}$ is the amplitude, $k^{in}$ is the wavenumber, and $d_{in}^j$ and $p_{in}^j$ are the unit vectors representing the directions of vibration and propagation, respectively, given by

$$d_{in} = (\cos \theta^{in}, -\sin \theta^{in}), \quad p_{in} = (-\sin \theta^{in}, -\cos \theta^{in}),$$

where $\theta^{in}$ is the incident angle. In eq.(1) and hereafter, the time factor $\exp(-i\omega t)$ is omitted.

The dynamic interaction of the incident wave with the crack generates a scattered wave, which is observed at the point $x$ at far field. We suppose that the crack is located on the plane $x_2 = 0$ prescribed in advance. The inverse problem considered here is, then, to determine the crack length from the scattered waves detected at far field. Our inversion method is based on the inverse Kirchhoff approximation, which will be described in the following.

2.2 Integral expression of scattered wave

The basic equation for the inverse Kirchhoff approximation is the integral expression of a scattered wave at far field. On the assumption that the crack is free of traction, the scattered displacement $u^{sc}_i$ at a point $x$ inside a domain $D$ may be written as follows:

$$u^{sc}_i(x) = -\int_S n_i(y) C_{jlmk} \frac{\partial}{\partial y_k} G_{im}(x, y)[u_j](y) ds_y, \quad x \in D$$

where $n_i(y)$ is the unit normal vector to the crack surface $S$, $C_{jlmk}$ are elastic moduli and $[u_j](y)$ denotes the crack opening displacement. In eq.(3), furthermore, $G_{im}(x, y)$ stands for the 2-D elastodynamic fundamental solution, which represents the displacement in the $i$-direction at the point $x$ due to the time harmonic force in the $m$-direction concentrated at the point $y$. 

Figure 1: Scattering of a plane incident wave by a crack.
For \( x_2 > y_2 \), the fundamental solution \( G_{im}(x, y) \) is given in the following integral form[6]:

\[
G_{im}(x, y) = \sum_{\alpha=L,T} \frac{i}{4\pi \mu} \left( \frac{k_{\alpha}}{k_T} \right)^2 \int_{-\infty}^{\infty} (1 - \zeta^2)^{-1/2} d_{\alpha}^T(\zeta) d_{m}^T(\zeta) e^{ik_{\alpha} p(\zeta)(x-y)} d\zeta \quad (4)
\]

where \( k_L \) and \( k_T \) are wavenumbers of P and S waves, respectively, and \( \mu \) is the shear modulus. Also, \( p(\zeta) \) and \( d^T(\zeta) \) are given by

\[
p(\zeta) = \begin{cases} \sqrt{1 - \zeta^2}, & \Im(\sqrt{1 - \zeta^2}) > 0 \end{cases}, \quad \text{for} \quad \alpha = L
\]

\[
d_L^T(\zeta) = \begin{cases} p(\zeta), & \text{for} \quad \alpha = L \\
\varepsilon_3 \times p(\zeta), & \text{for} \quad \alpha = T 
\end{cases}
\]

Assuming that the observation point \( x \) is far enough from the crack, we introduce the far field approximation \(|x - y| \approx |x| - \hat{x} \cdot y\) into eq.(4). The main contribution of the integral in eq.(4) comes from the saddle point \( \zeta = \zeta_s \), where \( \partial[p(\zeta) \cdot (x - y)]/\partial \zeta \big|_{\zeta = \zeta_s} = 0 \). The method of steepest descend provides an approximation to the integrals in eq. (4) as follows;

\[
G_{im}(x, y) \approx \sum_{\alpha=L,T} \sqrt{\frac{2}{\pi k_\alpha |x|}} e^{i(k_\alpha |x| - \pi/4)} d_{\alpha}^T(\zeta) D_{m}^T(\zeta) e^{-ik_{\alpha} p(\zeta)} y \bigg|_{\zeta = \sin \theta_x} \quad (7)
\]

where \( \theta_x = \arcsin(x_1/|x|) \) and

\[
D_{m}^T(\zeta) = \frac{i}{4\mu} \left( \frac{k_{\alpha}}{k_T} \right)^2 d_L^T(\zeta).
\]

Substituting eq.(7) into eq.(3), we have the integral expression for the scattered far field.

\[
u_{im}(x) \approx \sum_{\alpha=L,T} \sqrt{\frac{2}{\pi k_\alpha |x|}} e^{i(k_\alpha |x| - \pi/4)} d_{\alpha}^T(\zeta) e^{-ik_{\alpha} p(\zeta)} \Omega_\alpha(\theta_x, k_\alpha) \quad (9)
\]

where \( \Omega_\alpha \) represents the scattered pattern at far field, given by

\[
\Omega_\alpha(\theta_x, k_\alpha) = i k_\alpha C_{\alpha m} k_{\alpha} p(\zeta) D_{m}^T(\zeta) \int_S n_\alpha(y) e^{-ik_{\alpha} p(\zeta)} \Omega_\alpha(y) |u_j(y)| dy \bigg|_{\zeta = \sin \theta_x} \quad (10)
\]

The scattered pattern \( \Omega_\alpha \) involves the quantitative information on the crack, since \( \Omega_\alpha \) has the integration over the crack surface \( S \). In the following, we present a linear inversion method to determine the crack length from \( \Omega_\alpha \), which may be obtained from an ultrasonic experiment.

### 2.3 Kirchhoff inversion method

In order to linearize eq.(10), we introduce the Kirchhoff approximation into the crack opening displacement \( [u_j] \) in eq.(10). The crack opening displacement is then approximated by the sum of the incident wave and the reflected wave on the crack surface. For a line crack subjected to a plane incident wave, it follows that

\[
[u_j(y)] \approx f_j(\theta^{in}) e^{-ik_m \sin \theta^{in} y_1}, \quad y \in S \quad (11)
\]

where \( f_j \) is a function of the incident angle \( \theta^{in} \). As seen in eq.(11), no effect of crack edges is included in the crack-opening displacement introduced by the Kirchhoff approximation. It means that the approximation is valid only if the wavelength is smaller enough than the dimension of the crack. Substituting eq.(11) into eq.(10) yields the following expression:

\[
\Omega_\alpha(\theta_x, k_\alpha; \theta^{in}) = A_\alpha(\theta_x, \theta^{in}) k_\alpha \int_S e^{-i(k_{\alpha} \sin \theta_x + k_m \sin \theta^{in}) y_1} dy_1, \quad (12)
\]
Figure 2: Experimental setup.

where $A_\alpha(\theta_x, \theta^{in})$ is defined as

$$A_\alpha(\theta_x, \theta^{in}) = i f_j(\theta^{in}) C_{j2m}p_k(\zeta) \hat{D}_m^\alpha(\zeta)|_{\zeta = \sin \theta_x}. \tag{13}$$

In eq. (12), the argument $\theta^{in}$ is added to $\Omega_\alpha$ for the explicit expression of the incident angle.

We define a characteristic function $\Gamma$ as follows:

$$\Gamma(y_1) = \begin{cases} 1 & \text{for } y_1 \in S \\ 0 & \text{for } y_1 \notin S. \end{cases} \tag{14}$$

Using the characteristic function $\Gamma$, eq. (12) may be written as

$$\Omega_\alpha(\theta_x, k_\alpha; \theta^{in}) = A_\alpha(\theta_x, \theta^{in}) k_\alpha \int_{-\infty}^{\infty} \Gamma(y_1) e^{-i(k_\alpha \sin \theta_x + k^{in} \sin \theta^{in}) y_1} dy_1 \tag{15}$$

The integral in the above equation corresponds to the Fourier transform $\tilde{\Gamma}(K)$ of $\Gamma$. The characteristic function $\Gamma$ can, therefore, be obtained by using the inverse Fourier transform as follows:

$$\Gamma(y_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\Gamma}(K) e^{ik_\alpha y_1} dK$$

$$= \frac{1}{2\pi} A^{-1}_\alpha(\theta_x, \theta^{in}) \int_{-\infty}^{\infty} k^{-1}_\alpha \Omega_\alpha(\theta_x, k_\alpha; \theta^{in}) P(\theta_x, \theta^{in}) e^{ik_\alpha y_1} dK \tag{16}$$

where $P(\theta_x, \theta^{in}) = (\sin \theta_x + k^{in}/k_\alpha \sin \theta^{in})$. If the scattered patterns $\Omega_\alpha$ are detected over a wide range of frequency from experiment, then $\Gamma$, namely, the crack length can be reconstructed from eq. (16).

3. EXPERIMENT

3.1 Experimental setup

Ultrasonic experiments are carried out using a water immersed pulse-echo ultrasonic testing as shown in Fig. 2. The transducer is first excited by the ultrasonic pulser (JSR PR35) and radiates an incident ultrasonic wave into water. The incident wave is refracted at a water-solid interface and scattered by a crack.
inside the specimen. The scattered wave is received by the same transducer as the transmission transducer, amplified by the ultrasonic receiver (JSR PR35) and converted into digital data by the digital oscilloscope (HP 54600A). The transducer employed here is a piezoelectric broad band transducer with the nominal frequency of either 2MHz or 5MHz.

The specimens are two aluminum plates bonded together with artificial strip-shaped cracks as shown in Fig. 3. Cracks with lengths 2mm, 4mm and 6mm are prepared at the interface. The material properties of aluminum are given by the density of 2700 kg/m³, the P wave velocity of 6300 m/sec, the S wave velocity of 3100 m/sec.

The angle of the transducer is selected as $\theta_T = 19.7^\circ$ so that the incident angle of an S wave becomes 45°. In this case, no refracted P wave is generated in aluminum. Therefore, the pulse-echo configuration of an incident S wave and a scattered S wave is realized in the aluminum specimen.

3.2 Wave analysis based on linear system theory

As mentioned before, the inverse theory is formulated in a frequency domain. The transducer as well as other measurement devices has frequency dependent properties, which may distort the frequency characteristic of detected waves. Undesired frequency components may be removed by applying the linear system theory.

Suppose that the measurement system can be expressed using the linear system theory. Then we have the following relation between an input signal $I(t)$ and an output signal $O(t)$ in a time domain:

$$O(t) = I(t) \ast T(t) \ast W(t) \ast C_{WS}(t) \ast E(t) \ast C_{SW}(t) \ast W(t) \ast R(t)$$ (17)

where $\ast$ indicates the convolution with respect to time $t$, and $T$, $W$, $C_{WS}$, $E$, $C_{SW}$ and $R$ are the transmission functions of transducer output, water path, water to solid interface, interaction with crack in solid, solid to water interface and transducer reception, respectively. The Fourier transform of eq.(17) can be expressed as

$$O(\omega) = \tilde{I}(\omega)\tilde{T}(\omega)\tilde{W}(\omega)\tilde{C}_{WS}(\omega)\tilde{E}(\omega)\tilde{C}_{SW}(\omega)\tilde{W}(\omega)\tilde{R}(\omega)$$ (18)

where $\omega$ is the angular frequency and the bar indicates the Fourier transform of each transmission function. The physical meaning of the function $\tilde{E}(\omega)$ is the frequency response of a scattered wave by a crack in an infinite solid, which is proportional to the scattered pattern $\Omega_T$ defined by eq.(10).

In order to cancel unnecessary response functions except for $\tilde{E}(\omega)$, a signal from corner reflection is taken as a reference signal as shown in Fig. 4. The reference system consists of the same components as the
measurement system shown in Fig. 3 except for the corner reflection effect. Hence the reference signal can be written as

$$\hat{\theta}'(\omega) = \hat{I}(\omega)\hat{T}(\omega)W(\omega)\hat{C}_{WS}(\omega)\hat{E}'\hat{C}_{SW}(\omega)W(\omega)\hat{R}(\omega)$$  \hspace{1cm} (19)

where \( \hat{E}' \) represents the response of the corner reflection, which is independent of the frequency \( \omega \). From eqs.(18) and (19), we have

$$E(\omega) = E' \frac{O(\omega)}{\hat{O}'(\omega)}$$  \hspace{1cm} (20)

Since \( E \) is proportional to \( O_T \), \( E \) is substituted into eq.(16) with \( \alpha = T \) and \( \theta_2 = \theta_{in} = 45^\circ \) to reconstruct the characteristic function \( \Gamma \).

4. RESULTS

Figs. 5 (a), (b) and (c) show the original waveforms scattered by cracks with lengths 2mm, 4mm and 6mm, respectively. These waves are detected by the transducer with the nominal frequency of 2MHz. As the crack length is larger, two diffracted waves from crack tips separate clearly.

Figs. 6 show the frequency spectra corresponding to Figs. 5. The interference of two diffracted waves from crack tips generates periodic oscillation in the frequency spectra, which can also be used to determine the crack length[5].

In the Kirchhoff inversion method, all frequency components from 0 to \( \infty \) are theoretically required as shown in eq.(16). In experiment, however, it is impossible to get such broad band frequency components. As shown in Fig. 6, the ultrasonic wave has relatively narrow band frequency components. For wave data detected by the 2MHz transducer, therefore, frequency components from 0.5MHz to 3.5MHz are used in the inversion analysis.

Fig. 7 shows the reconstructed characteristic functions \( \Gamma \) for the cracks with lengths 2mm, 4mm and 6mm. The solid lines are reconstructed functions and the dashed lines are exact solutions defined by eq.(14). Agreement of the reconstructed function with the exact solution is not so good. Since sharp minimum points are seen at the crack tips, however, it is possible to determine the location and size of the crack.

Fig. 8 shows the time domain waveforms detected by the 5MHz transducer for the cracks with lengths 2mm, 4mm and 6mm. The diffracted waves from crack tips show clearer separation than the waves detected by the 2MHz transducer do. Fig. 9 represents the characteristic functions reconstructed from wave data shown in Fig. 8. In this case, the frequency components within the range from 1MHz to 8MHz are used in the inversion analysis. In the same way as in Fig. 7, the reconstructed functions \( \Gamma \) show the sharp variations at the crack tips, so that we can know where the crack exists.

Finally, we summarize several reasons for the disagreement between theory and experiment.

- In the theory, a plane incident wave is assumed. However, the incident wave emitted from the transducer may be a spreading beam like a spherical wave.

- The Kirchhoff approximation is valid only in a high frequency range, where the effect of diffraction can be neglected. The frequency used in the present inversion process, however, involves a middle frequency range around \( ak_T = 1 \), where the accuracy of the Kirchhoff approximation is not good enough.

- In the inversion method based on the inverse Fourier transform, all frequency components from 0 to \( \infty \) are ideally required as shown in eq.(16). In experiment, however, even a broad band transducer can detect a waveform with a limited frequency range. Therefore the stepwise characteristic function can not be reconstructed perfectly.
Figure 5: Time domain waveforms detected by 2MHz transducer for the cracks with lengths (a) 2mm, (b) 4mm and (c) 6mm.

Figure 6: Fourier amplitudes of scattered waves by the cracks with lengths (a) 2mm, (b) 4mm and (c) 6mm.
Figure 7: Reconstructed (solid lines) and exact (dashed lines) characteristic functions for cracks with lengths (a) 2mm, (b) 4mm and (c) 6mm. Scattered waves are detected by 2MHz transducer.
Figure 8: Time domain waveforms detected by 5MHz transducer for the cracks with lengths (a) 2mm, (b) 4mm and (c) 6mm.

Figure 9: Reconstructed (solid lines) and exact (dashed lines) characteristic functions for cracks with lengths (a) 2mm, (b) 4mm and (c) 6mm. Scattered waves are detected by 5MHz transducer.
5. CONCLUSION

The inverse Kirchhoff approximation was applied to ultrasonic data scattered by a crack in an aluminum specimen. Although the reconstructed characteristic functions are not in perfect agreement with the exact ones, sharp minimum points appeared at crack tips. Hence we could determine the location and size of the crack.

In this paper, we applied the inversion method to a single line crack in a solid. The present method can also be applicable to multiple cracks and a planar crack distributed in a prescribed plane.

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REFERENCES


