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Computational Analysis of Currents Generated in Kojima Lake Based on Updated Topographical Data

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Topographical data of Kojima Lake have been updated by applying data obtained after dredging. Together with tide level of Kojima Bay and water levels of Kurashiki River and Sasagase river, the information was incorporated in computational analysis of unsteady flows generated in the lake when the water was discharged from Kojima Lake to Kojima Bay. A finite element method was applied to equations governing unsteady flows, and the transition of discharge was obtained.

Key words: *water environment, finite element method, numerical simulation*

1 INTRODUCTION

Kojima lake is an artificial lake created by separating a part of Kojima Bay from the rest. Kurashiki river and Sasagase River, which flow into Kojima Lake, are primary sources of the water in the lake. There are 6 gates that connect Kojima Lake and Kojima Bay, and they are opened when a discharge of water Kojima Lake to Kojima Bay is necessary in order to control the water level of the lake. Such a discharge of the water generates an unsteady flow in Kojima Lake. A finite element method was applied to analyze the unsteady flows numerically, and the transition of discharge was obtained.

In the following sections, techniques to analyze flows generated in Kojima Lake as well as numerical results are introduced. In Section 2, a technique to update topographical data of Kojima Lake is illustrated with an example. In Section 3, numerical techniques to analyze unsteady flows are described. In Section 4, some numerical results are introduced.

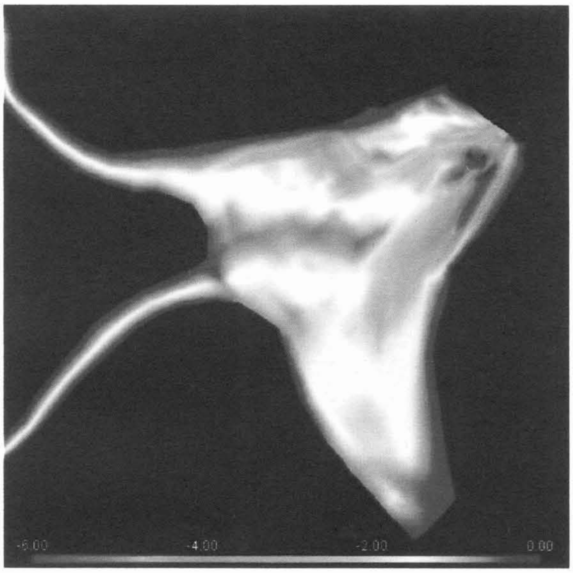

Figure 1: Bottom topography of Kojima Lake based on data reported in 1989 (Okayama Prefecture Okayama Development Bureau).

2 BOTTOM TOPOGRAPHY OF KOJIMA LAKE

In analysis of flow in the water environment, data concerning the bottom topography often become indispensable. Figure 1 shows the bottom topography of Kojima Lake based on data reported in 1989. Kojima Lake has constantly been dredged, and it becomes necessary to update the data. Figures 2 shows the bottom topography of parts that have been measured recently. The information has been introduced to update the data shown in Figure 1, and the result is shown in Figure 3.

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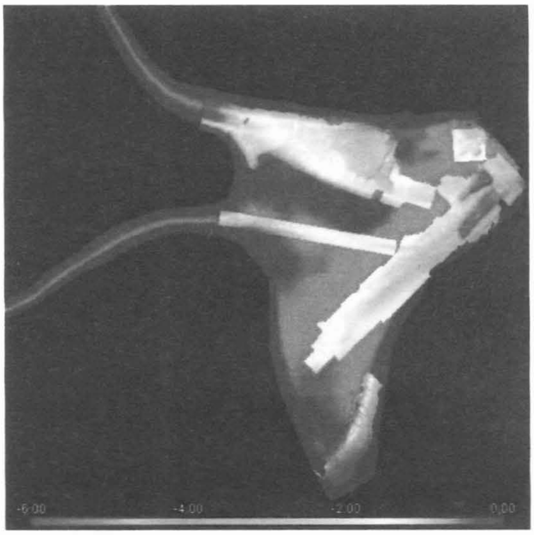

Figure 2: Bottom topography of some parts of Kojima Lake based on data supplied by Chugoku-Shikoku Agricultural Administration Office.

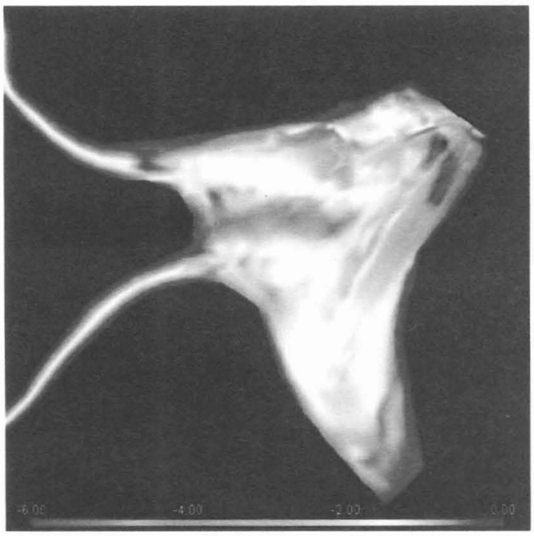

Figure 3: Updated bottom topography of Kojima Lake.

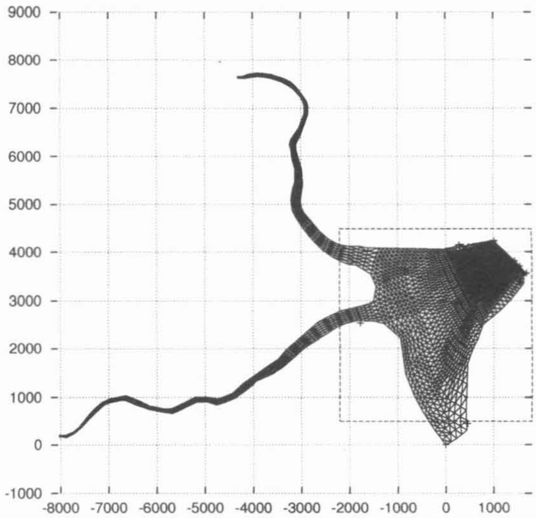

Figure 4: Finite elements in the entire region. There are 3636 elements and 1966 nodes.

3 DESCRIPTION OF FINITE ELEMENT ANALYSIS

A finite element method was applied to the system consisting of momentum equations and a continuity equation:

$$\begin{aligned}
\frac{\partial M}{\partial t} &= -g\,(h+\zeta)\,\frac{\partial \zeta}{\partial x} + A\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) - \frac{\tau_x}{\rho_0}, \\
\frac{\partial N}{\partial t} &= -g\,(h+\zeta)\,\frac{\partial \zeta}{\partial y} + A\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - \frac{\tau_y}{\rho_0}, \\
\frac{\partial \zeta}{\partial t} &= -\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}.
\end{aligned} \tag{1}$$

Here, $z = \zeta$ and $z = -h$ correspond to the water surface and the bottom of the lake, respectively. Let u and v denote x-component and y-component of the velocity, respectively. M and N are defined by

$$M = \int_{-h}^{\zeta} u\,dz, \quad N = \int_{-h}^{\zeta} v\,dz.$$

ρ_0 is a constant that represents the density. τ_x and τ_y are given by

$$\tau_x = \frac{\rho_0 \gamma^2 \sqrt{M^2+N^2}}{(h+\zeta^2)} M, \quad \tau_y = \frac{\rho_0 \gamma^2 \sqrt{M^2+N^2}}{(h+\zeta^2)} N.$$

In order to solve the system of partial differential equations (1) numerically, we set $g = 9.81$ and $A = 0.001$. Figures 4 and 5 show the division of the region into triangular elements. There are 3636 elements and 1966 nodes in the division. The part enclosed by the box of Figure 4 is shown in Figure 5.

We analyzed unsteady flows generated in Kojima Lake when the gates were opened to discharge the water from the lake into Kojima Bay from 2:40 am to

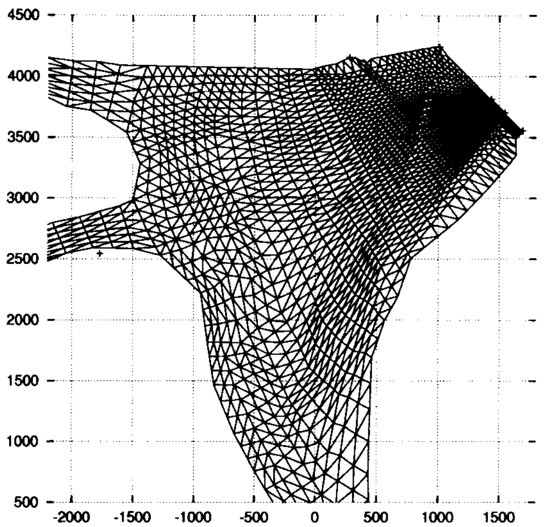

Figure 5: Finite elements enclosed by the box of Figure 4.

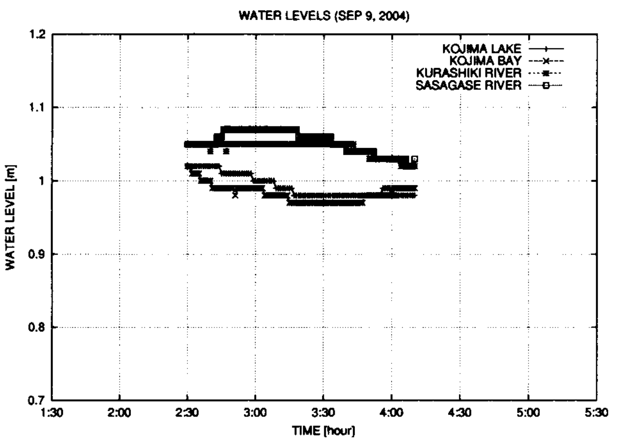

Figure 6: Transition of water levels on September 9, 2004.

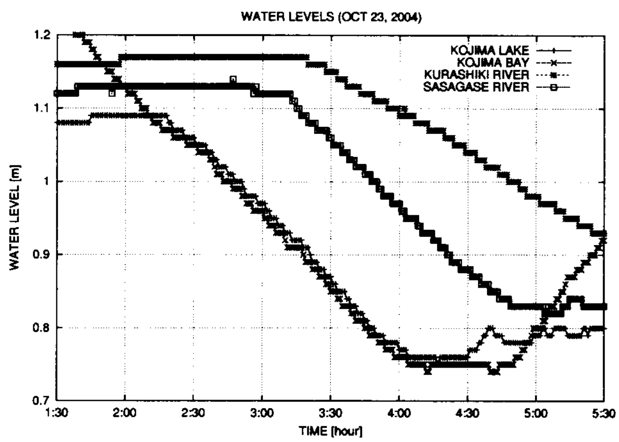

Figure 7: Transition of water levels on October 23, 2004.

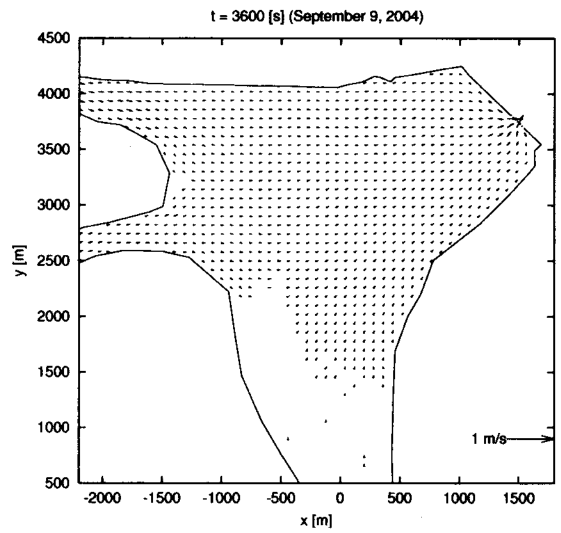

Figure 8: Velocity vectors at one hour after the gates were opened on September 9, 2004.

4:00 am (GMT) on September 9, 2004 and from 2:10 to 4:40 am (GMT) on October 23, 2004. Figures 6 and 7 show the transition of water levels of Kojima Lake, Kurashiki River, and Sasagase River, and the tide level of Kojima Bay on September 9, 2004 and October 23, 2004, respectively. The water levels of Kurashiki River and Sasagase River, and the tide level of Kojima Bay were introduced into computational analysis of the unsteady flows as boundary conditions for the system of partial differential equations.

4 NUMERICAL RESULTS OF FINITE ELEMENT ANALYSIS

Numerical results concerning the unsteady flows generated in Kojima Lake on September 9, 2004, and October 23, 2004 are shown in Figures 8 - 11. Figures 8 and 9 show the distribution of the velocity vectors at one hour after the gates were opened on September 9, 2004, and October 23, 2004, respectively. Figures 10 and 11 show the actual transition of water level of Kojima Lake on September 9, 2004, and October 23, 2004, respectively, and numerical results that simulate the transition. A reasonable agreement between the numerical results and the actual transition is shown in those figures.

In order to simulate the transition of the total discharge of water through the gates, the discharge through the gates was integrated with respect to time. Suppose that $(x, y) = (x_{i,0}, y_{i,0})$ and $(x, y) = (x_{i,1}, y_{i,1})$ are the end points of the ith gate $(i = 1, 2, \ldots, 6)$. Let

$$\gamma_i(s) = (1 - s)(x_{i,0}, y_{i,0}) + s(x_{i,1}, y_{i,1})$$

Then the temporal rate of the outflow through the

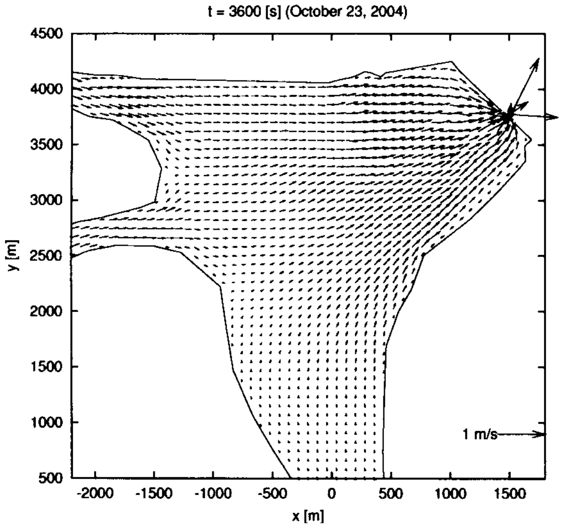

Figure 9: Velocity vectors at one hour after the gates were opened on October 23, 2004.

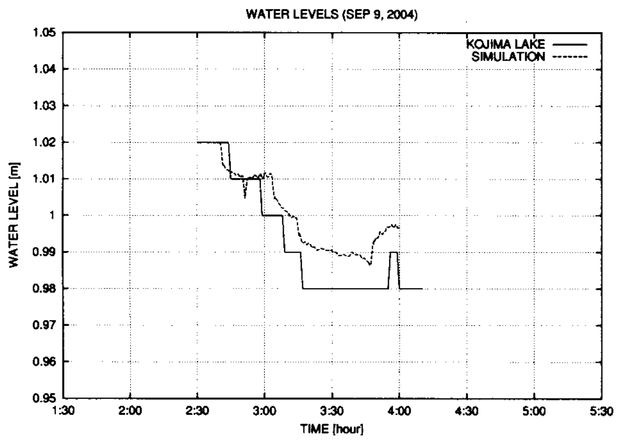

Figure 10: Transition of water level of Kojima Lake on September 9, 2004.

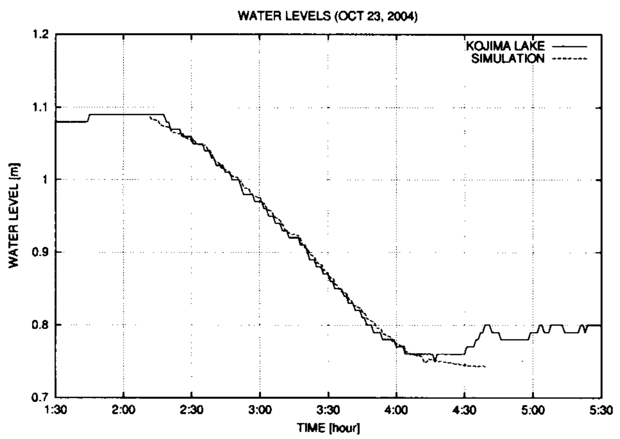

Figure 11: Transition of water level of Kojima Lake on October 23, 2004.

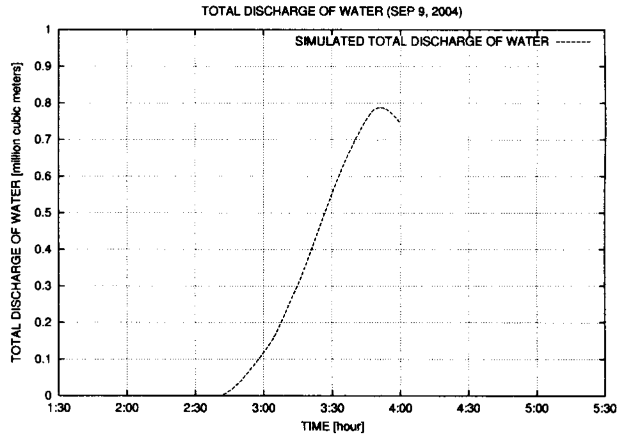

Figure 12: Discharge of water on September 9, 2004.

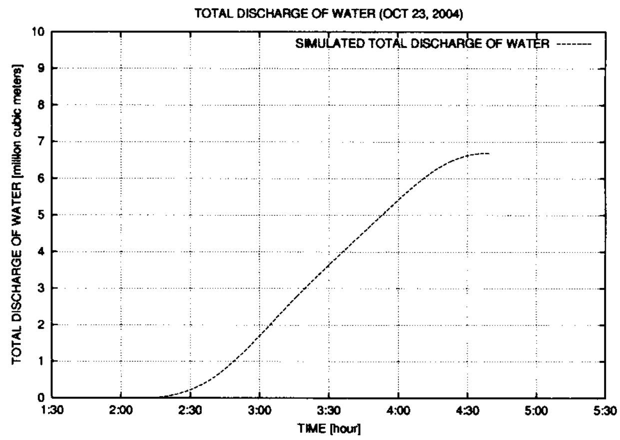

Figure 13: Discharge of water on October 23, 2004.

gates is given by

$$\sum_{i=1}^{6} w_i \int_0^1 \left(\int_{-h}^{\eta} \mathbf{v} \cdot \mathbf{n}_i \, dz \right) ds \qquad (2)$$

where $(x, y) = \gamma_i(s)$. The constant w_i is the width of the ith gate:

$$w_i = \sqrt{(x_{i,1} - x_{i,0})^2 + (y_{i,1} - y_{i,0})^2},$$

the vector $\mathbf{n}_i$ is a unit vector normal to the ith gate

$$\mathbf{n}_i = \frac{1}{w_i} (x_{i,1} - x_{i,0}, y_{i,1} - y_{i,0}),$$

and the vector $\mathbf{v}$ is the velocity vector. The trapezoidal rule is applied to the expression (2) to obtain the discharge from Kojima Lake to Kojima Bay. Figures 12 and 13 show the transition of the total discharge due to the unsteady flow generated on September 9, 2004 and October 23, 2004, respectively.

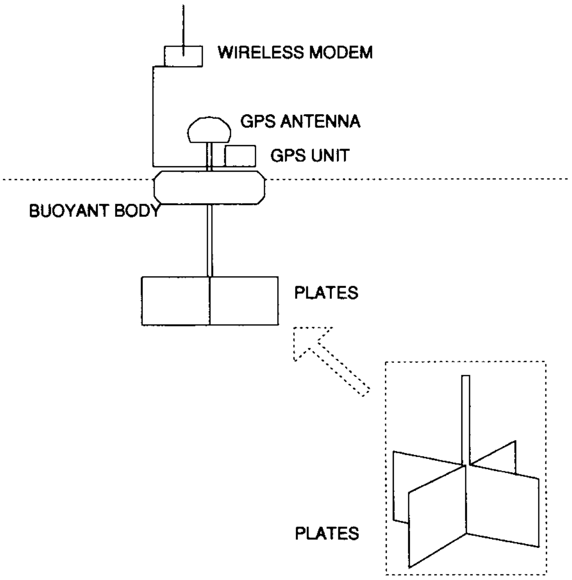

Figure 14: Illustration of the GPS-float.

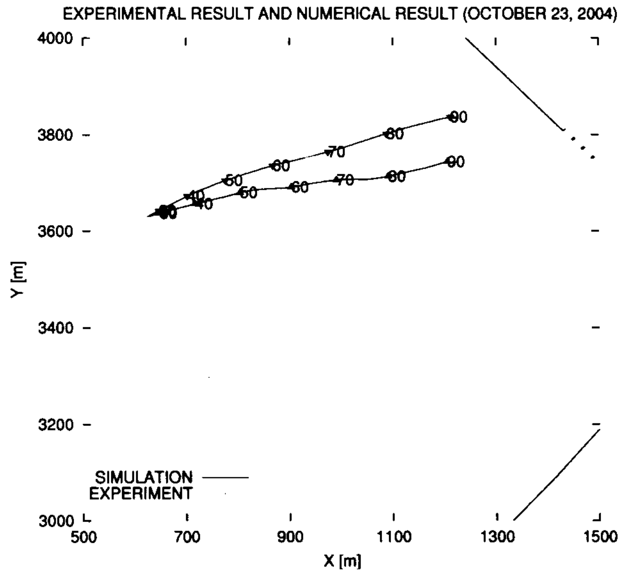

Figure 15: Experimental results and numerical results concerning the motion of the GPS-float (October 23, 2004).

Numerical simulation of the motion of a float

We conducted an experiment to study flows generated in Kojima Lake using a float equipped with a GPS unit, which we call the GPS-float. The GPS-float is designed to travel over the surface receiving the fluid resistance on a pair of rectangular plates attached underneath the surface. The GPS-float is illustrated in Figure 14. The fluid resistance exerted on the plates attached to the GPS-float can be evaluated in terms of the fluid velocity. Thus, once the velocity of flow is obtained, the motion of the GPS-float can be simulated by solving its momentum equation Watanabe 2004:

$$\begin{aligned} M\ddot{x} &= \frac{C_D S \rho_0}{2}(u-\dot{x})\sqrt{(u-\dot{x})^2+(v-\dot{y})^2} \\ &\quad -F\dot{x}\sqrt{(\dot{x})^2+(\dot{y})^2} \\ M\ddot{y} &= \frac{C_D S \rho_0}{2}(v-\dot{y})\sqrt{(u-\dot{x})^2+(v-\dot{y})^2} \\ &\quad -F\dot{y}\sqrt{(\dot{x})^2+(\dot{y})^2} \end{aligned}$$

We solved the momentum equation to simulate the motion of the GPS-float setting

$$C_D = 1.15, \quad S = 1.75, \quad \rho_0 = 1000.0,$$
$$M = 13.0, \quad F = 20.0.$$

Figure 15 shows some results of the numerical simulation.

5 CONCLUSION

Flows generated in Kojima Lake have been studied by analyzing equations governing their dynamics. (Watanabe 1999, 2000 (1), (2), 2002 (1), (2), Watanabe, et al. 2001, Numaguchi *et al.* 2004, Watanabe 2004, Watanabe *et al.* 2004, Watanabe *et al.* 2005, Watanabe *et al.* submitted). They have also been studied experimentally. The GPS-float has been developed to study flow in the water environment experimentally (Watanabe 1999, 2000 (2), 2002 (1), (2), Watanabe *et al.* 2001, Watanabe *et al.* 2003, Numaguchi *et al.* 2004, Watanabe 2004, Watanabe *et al.* 2004, Watanabe *et al.* 2005, Watanabe *et al.* submitted). It is a float equipped with a GPS unit. While it travels on the surface of water under the influence of flow, the GPS unit analyzes its position, and the spatial and temporal data that specify its motion are recorded to be analyzed. One can simulate the motion of the GPS-float by solving its momentum equation provided the fluid resistance is specified, and, the fluid resistance can be given in terms of the velocity of the flow (Watanabe 1999, 2000 (2), 2002 (1), (2), Watanabe, et al. 2001, Numaguchi *et al.* 2004, Watanabe 2004, Watanabe *et al.* 2004, Watanabe *et al.* 2005, Watanabe *et al.* submitted).

In order to asses the water quality in the environment, it is important to understand the circulation of water. Here we analyzed the discharge from the lake incorporating the actual data such as topographical data as well as data concerning water levels and tide levels into computational analysis. Figures 10 shows that approximately 30,000 m^3 of water was discharged from Kojima Lake into Kojima Bay on September 9, 2004, whereas 300,000 m^3 of water was discharged on

October 23, 2004.

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