A Farmer's Subjective Equilibrium under the Opening of a Water Market

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Summary

To use water resources efficiently, it becomes necessary to introduce a water market which connects the agricultural sector with the non-agricultural sector.

In this paper, I will discuss about a farmer's subjective equilibrium associated with water and other resources use, and the efficiency of water and other resources use when a water market is introduced.

From the standpoint of minimizing social losses associated with water and other resources use, we can say that introduction of a water market is beneficial to society. However, water pricing policies will give big impacts not only on water resources use but on other resources use. Therefore, when we discuss about making a water market, we must pay attention to introducing what kinds of water pricing policies.

Introduction

From the Microeconomic Theory, we know that to use water resources efficiently, (1) if it is directly consumed, each consumer's marginal utility of water must be equal, or (2) if it is used as a productive input, each user's marginal value product of water must be equal, or (3) if it is allocated between direct consumptive use and productive input use, the marginal utility of water in consumption must be equal to the marginal value product of water in production.

But the real situation in water resources' use is very far from the ideal situation presented in the Microeconomic Theory. To achieve the ideal situation, a water market which connects each water user has to exist and work well. However, because of the barriers which social institutions raise the function of a water market is restricted. For example, water users hold water rights and the water rights are strongly protected by the Law and traditional social institutions. Therefore, the transfer of water rights or the selling and buying of water rights is very difficult. Even though a water market could exist under these restricted conditions, each water user's marginal valuation of water will not be equal and usually a big gap will exist between each water user's marginal valuation of water.

If we stand on the idea that water rights are not proprietary rights but rather rights to use water and belong to society, to solve this kind of disequilibrium (or unbalance), it is necessary for us to reallocate water resources. In short, it is necessary to transfer one part of former's water rights to the latter until their marginal valuation of water becomes equal. But this reallocation might cause income reallocation between these water users. If this situation happened and if this reallocation was done without introducing a compensation mechanism, the former's welfare level would decrease and the latter's welfare level will increase compared to the situation where reallocation had not been done. If society could get net benefit through this reallocation (i.e., the latter's benefit was greater than that of the former's) and this net benefit could be reallocated between them, both parties would be better off compared to the pre-reallocation.

However it is very difficult to introduce such a compensation mechanism. From which gainers
and how much of a percentage of the gain should be reallocated among the losers? Persons
who think they might lose by this reallocation would have a strong incentive to prevent
the reallocation. If this situation happened, even though the reallocation was socially desirable,
it could not be done. Therefore, to introduce a water market smoothly, we have to pay much
attention to how a water market can be introduced without decreasing the welfare level of the
persons who will lose from its introduction.

I think that the way which has the highest possibility for smoothly introducing a water
market is to introduce a water market with a guarantee of the vested right associated with
water resources' use for the persons who will lose by the introduction. To put it in the
concrete, through the amendment of the institutions associated with water resources' use, it
is necessary to allow the persons to be able to sell the water rights which they have held or
to be able to sell water while keeping their water rights. Though, in Japan, these behaviors
are prohibited by "RIVER CONTROL LAW (1964)". In this way, their welfare associated
with water use will decrease but they could recover this decrease in welfare by selling water
rights or selling water while keeping their water rights. If the revenue from selling water
rights or selling water did not recover the decrease in their welfare associated with the water
use decrease, they could maintain their current welfare level without selling water rights or
without selling water. In other words, only when their users believe that at least they will
not be worse off, they will sell their water rights or water while keeping their water rights.

When the water market is introduced, between which parties will water resources be
transferred?. Generally speaking, in the current situation one part of the water resources
which are used in the agricultural sector will be transferred to the non-agricultural sector. Based
on the introduction of a water market, let us assume that one part of the water resources
will be transferred from the agricultural sector to the non-agricultural sector and that farmers
are at least guaranteed to be able to keep the vested right which they have held before the
introduction of the water market (i.e., they can sell their water rights or they can sell water
while keeping their water rights). Under these conditions, when the water market is intro-
duced, will farmers choose to sell their water rights or to sell water while keeping their
water rights?. Which way's possibility is high?.

The main reason why current farmers enjoy an abundance of water depends on the diligent
efforts that their ancestors spent on developing water resources. Sometimes farmers' ancestors
accomplished this at the risk of their lives because in those days the technological level of
civil engineering was very low. For example, in Japan, agricultural village communities are
like settled societies. In other words, the same families have been living in the same village
communities for several tens of generations. Moreover, each village community has the history
book of that village. Therefore, the current village residents know exactly how their ancestors
created the water resources which are now available. Should farmers be threatened with
losing their water rights, they would remember their ancestors' hard efforts, will strongly
become attached to their water rights and therefore their subjective evaluation of the water
rights will be quite high.

Consequently, it is reasonable to think that farmers will opt to sell water while keeping
their water rights. As farmers have a strong incentive to averse risk, they may be unwilling
to risk what might happen in the future if they sell out their water rights. Farmers may
consider that after having sold out their water rights, a heavy dry year might befall them or
in the future they may wish to increase agricultural production or to introduce new varieties
that might need increased amounts of water. Thus, even if a water market is introduced which
guarantees farmers' vested rights in water, generally speaking, I think the possibility for them
to sell water while keeping their water rights is rather high.

I will consider more concretely a hypothetical situation whereby farmers sell water while
keeping their water rights. In Japan, in the real situation associated with water resources' use
in the agricultural sector, a water users' community ("Tochi Kaııyō Ku" in Japanese) is the base unit organization and it holds the water rights as a whole and the water rights are divided and each division of the water rights is attached to each member farmers' land and each member farmer pays water charges to the community for using water. Therefore, in this hypothetical situation, water will be sold through the water users' community. Hence, we can imagine that the quantity of water which each farmer is using before the introduction of a water market will be guaranteed as the vested right of each farmer and the difference between that quantity and the quantity of water which each farmer will use after the introduction of a water market will be recognized as each farmer's selling a quantity of water and each farmer will receive the money associated with the selling of water from the water users' community.

The objectives of this paper are ① to analyze, when a water market which allows farmers to sell water while keeping their water rights is introduced, what a farmer's subjective equilibrium conditions associated with resources' use will be, and ② to explain, as the result of the introduction of the water market, how the efficiency of water resources' use and another resources's use will be changed.

Materials and Methods

Before the analysis, I will establish four hypothetical situations relevant to farmers' water use situations. The first two cases are where ① the water which the farmer uses is bought only from the water users' community to which he belongs, and ② the water which the farmer uses is partly bought from the water users' community and partly pumped up from underground. According to each of these two cases, I will set two additional cases where ① the price (or charge) of water which the farmer pays to the water users' community is determined by per unit acre of irrigated land, and ② the price (or charge) of water which the farmer pays to the water users' community is determined by per unit quantity. As the results, we therefore have four cases showed in Table 1.

<table>
<thead>
<tr>
<th>Sources of water</th>
<th>Water price systems</th>
</tr>
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<tbody>
<tr>
<td>Case ①</td>
<td>①−①</td>
</tr>
<tr>
<td>Case ②</td>
<td>①−②</td>
</tr>
</tbody>
</table>

Table 1. Four types of farmers' water use situations

I will make some assumptions for the convenience of the analysis.

[Assumptions]
(A-1): Climate conditions are normal and stable
(A-2): A farmer is a profit maximizer
(A-3): A farmer produces one agricultural product
(A-4): A farmer is a price taker in input factors' (or resources') markets
(A-5): A farmer is a price taker in the product's market
(A-6): A farmer's production function is as follows:

\[ Q = F[K, L, W] \]

\( Q \): the output
\( P \): price of the output
\( F(\cdot) \): this function is assumed to be strictly concave (associated with each input factor) and each input factor works co-operatively for the output production
\( K \): capital goods
\( P_k \): price of the capital goods
\( L \): land
\( P_i \): land price per unit acre
\( W \): the quantity of water which is actually available for the plant in the root zone

(A-7): \( A = B + X \)

\( A \): applied water
\( B \): the quantity of water bought from the water users’ community
\( X \): the quantity of water pumped up from the underground

Due to evaporation and percolation into the underground, etc., the plant can not utilize 100% of \( A \). But, through capital investment in water saving, we can increase the utilization rate of \( A \). Thus, I specify the relationships between \( A \), \( W \) and \( C \) as follows:

\[
W = a(C) \cdot A
\]

\[0 < a \leq 1, \ a(0) > 0, \ a(\infty) = 1, \ \frac{d a}{dC} = a' > 0 \]

\[\frac{d^2 a}{dC^2} = a'' < 0, \ \lim_{C \to a} a' \to \infty\]

\( C \): capital investment in water saving

Before the Opening of a Water Market

1) In CASE (1-1)

Let \( P_e \) be the price of water as determined by per unit acre of irrigated land which the farmer pays to the water users’ community (for short, W.U.C.). Therefore, the total payment to the W.U.C. associated with the water use will be \( P_e \cdot L \). Where \( L \) is the total land area which is irrigated by the water bought from the W.U.C.. The farmer’s profit function \( (\pi_1) \) is

\[(1) \quad \pi_1 = P \cdot Q - P_k \cdot K - P_t \cdot L - P_k \cdot C - P_e \cdot L \]

From the necessary conditions for profit maximization, the farmer’s subjective equilibrium conditions associated with each resources’ use are

\[(2) \quad P \cdot F_k = P_k \]
\[(3) \quad P \cdot F_t = P_t + P_e \]
\[(4) \quad P \cdot \left( \frac{\partial F}{\partial W} \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial B} \right) = P \cdot F_w \cdot a = 0 \]
\[(5) \quad P \cdot \left( \frac{\partial F}{\partial W} \cdot \frac{\partial W}{\partial C} \right) = P \cdot F_w \cdot a \cdot B = P_k \]

From equations (4) and (5)

\[(6) \quad \left| \begin{array}{c}
\frac{\partial W}{\partial C} \\
\frac{\partial W}{\partial A} \\
\frac{\partial W}{\partial L}
\end{array} \right| = \left| \begin{array}{c}
P_k \\
0 \\
0
\end{array} \right| = \infty \]

In these equations, \( \frac{\partial F}{\partial K} \) is substituted by \( F_k \), \( \frac{\partial F}{\partial L} \) by \( F_t \) and \( \frac{\partial F}{\partial W} \) by \( F_w \). The left hand side of each equation from (2) to (6) indicates the marginal value product (for short, M.V.P.) of \( K \), M.V.P. of \( L \), M.V.P. of \( B \), M.V.P. of \( C \) and the marginal rate of substitution (for short, M.R.S.) of \( C \) to \( A \), respectively. \( K^*, L^*, B^* \) and \( C^* \), which are the optimal input of each resource respectively, are simultaneously determined by equations (2) to (5).

The interesting points in these subjective equilibrium conditions are that (1) the land price which the farmer conceives of subjectively (for short, Farmer’s Subjective Land Price) is not
\(P_i\), but \((P_r + P_o)\). Consequently, the farmer begins to use too little land compared to the perfectly competitive land market situation, because the Farmer’s Subjective Land Price (or the M.V.P. of land on his subjective equilibrium) is higher than the land market price. Therefore, when we compare this situation to the perfectly competitive land market situation, land resources will be used inefficiently. \(2\) \(W\) will be inputted until marginal physical product (for short, M.P.P.) of \(W\) approaches zero (‘\(\cdot\)’ in equation (4), \(a > 0\) and \(P > 0\)) or in other words, until the total output associated with \(W\) arrives at the maximum. This means that the farmer uses water as if it were a free goods. \(3\) From equation (5), we know \(a'(C^*) = \infty (‘\(\cdot\)’ \(P > 0, B > 0, P_k > 0\) and \(F_w = 0\)). Therefore, \(C^*\) is zero. In other words, the farmer does not invest in saving water.

2) In CASE \(I – 2\)

Let \(P_b\) be the price of water as determined by per unit quantity which the farmer pays to the W.U.C.. In this case, the farmer’s profit function \((\pi_t)\) is

\[
\pi_t = P \cdot Q - P_k \cdot K - P_i \cdot L - P_b \cdot C - P_b \cdot B
\]

From the necessary conditions for profit maximization, the farmer’s subjective equilibrium conditions associated with resources’ use are

\[
P \cdot F_k = P_k
\]

\[
P \cdot F_l = P_l
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial B} = P \cdot F_w \cdot a = P_b
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial C} = P \cdot F_w \cdot a' \cdot B = P_k
\]

From equations (10) and (11)

\[
\left| \frac{\partial W}{\partial C} \right| = \left| \frac{P_k}{P_b} \right|
\]

The left hand side of each equation from (8) to (12) indicates the M.V.P. of \(K\), M.V.P. of \(L\), M.V.P. of \(B\), M.V.P. of \(C\) and the M.R.S. of \(C\) to \(A\), respectively. \(K^{**}, L^{**}, B^{**}\) and \(C^{**}\), which are the optimal input of each resource respectively, are simultaneously determined by equations (8) to (11).

The interesting points relating to these subjective equilibrium conditions when we compare these to the results of CASE \(I – 1\) are that (1) the Farmer’s Subjective Land Price (or the M.V.P. of land on his subjective equilibrium) is equal to the land market price. Consequently, land resources are used efficiently as in a perfectly competitive land market situation. \(2\) \(W\) is not inputted until M.P.P. of \(W\) approaches zero (‘\(\cdot\)’ from (10), \(F_w > 0\)). This result means that the farmer no longer uses water as if it were a free goods. \(3\) From equation (11), we know \(0 < a'(C^{**}) < \infty\). Therefore, \(C^{**}\) is greater than zero. In other words, the farmer begins to invest in saving water.

3) In CASE \(II – 1\)

The cost function \((Z)\) for pumping up \(X\) is

\[
Z = \bar{Z} + \tilde{Z}(X), \quad \frac{dZ}{dX} = Z' > 0, \quad \frac{d^2Z}{dX^2} = Z'' > 0
\]

where \(\bar{Z}\) is fixed cost and \(\tilde{Z}\) is variable cost. The farmer’s total land area \((L)\) which is
irrigated is \( L = L_1 + L_2 \) where \( L_1 \) is the land area which is irrigated by \( B \), i.e., \( L_1 = \frac{B}{A} \cdot L \) and \( L_2 \) is the land area which is irrigated by \( X \), i.e., \( L_2 = \frac{X}{A} \cdot L \). Thus the farmer's profit function (\( \pi_3 \)) is

\[
\pi_3 = P \cdot Q - P_k \cdot K - P_t \cdot L - P_e \cdot L_1 - P_s \cdot C - Z
\]

From the necessary conditions for profit maximization, the farmer's subjective equilibrium conditions associated with each resources' use are

\[
P \cdot F_k = P_k
\]

\[
P \cdot F_t = P_t + P_e \cdot \frac{B}{A}
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial A} = P \cdot F_w \cdot a = P_e \cdot \frac{L}{A} \cdot \frac{X}{A}
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial X} = P \cdot F_w \cdot a = Z' - P_e \cdot \frac{L}{A} \cdot \frac{B}{A}
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial C} = P \cdot F_w \cdot A \cdot a' = P_k
\]

From equations (17), (18) and (19)

\[
\left| \frac{\partial W}{\partial C} \right| = \left| \frac{P_k}{P_e \cdot L/A \cdot X/A} \right| \quad \text{or} \quad \left| \frac{\partial W}{\partial A} \right| = \left| \frac{P_k}{Z' - P_e \cdot L/A \cdot B/A} \right|
\]

The left hand side of each equation from (15) to (20) indicates the M.V.P. of \( K \), M.V.P. of \( L \), M.V.P. of \( B \), M.V.P. of \( X \), M.V.P. of \( C \) and the M.R.S. of \( C \) to \( A \), respectively.

From equations (17) and (18), the M.V.P. of \( B \) should be equal to the M.V.P. of \( X \). In another way, we can show the right hand side of equation (17) as \( P_e \cdot \frac{\partial L_1}{\partial B} \). Here, \( \frac{\partial L_1}{\partial B} \) shows how much the land area irrigated by \( B \) will increase when the farmer increases the use of \( B \) by \( \Delta B \). Hence, \( P_e \cdot \frac{\partial L_1}{\partial B} \) is the marginal increase of payment to the W.U.C. which is associated with the marginal increase of \( B \). Therefore, we can call the right hand side of equation (17) the marginal cost associated with the change of \( B \) (for short, M.C._b). In the right hand side of equation (18), the second term \((- P_e \cdot \frac{L}{A} \cdot \frac{B}{A})\) is derived from \( P_e \cdot \frac{\partial L_1}{\partial X} \).

The term \( \frac{\partial L_1}{\partial X} \) shows how much the land area irrigated by \( B \) will decrease when the farmer increase the use of \( X \) by \( \Delta X \). Therefore, \( P_e \cdot \frac{\partial L_1}{\partial X} \) is the marginal decrease of payment to the W.U.C. which is associated with the marginal increase of \( X \). \( Z' \) reflects marginal increase of pumping costs associated with the marginal increase of \( X \). Hence, we can call the right hand side of equation (18) the net marginal cost associated with the change of \( X \) (for short, N.M.C._x).

\( K^*, L^*, B^*, X^* \) and \( C^* \), which are the optimal input of each resource respectively, are simultaneously determined by equations (15) to (19).

The interesting points relating to these subjective equilibrium conditions when we compare these to the results of CASE \( \text{I} \) to \( \text{I} \) are that \( \text{I} \) the Farmer's Subjective Land Price is
higher than the land market price, but it is lower than that of CASE I—I (\(P_t + P_e \cdot \frac{B}{A} < P_t + P_e\)). Therefore, even though land resources will be used inefficiently compared to the perfectly competitive land market situation, the degree of inefficiency of land use is smaller than that of CASE I—I. ② From equation (17), we know \(F_w\) is positive. That is to say, the farmer does not input \(W\) until M.P.P. of \(W\) becomes zero (or the total output associated with \(W\) is maximized). The reason is that as the farmer evaluates the unit quantity of \(B\) by M.C., he no longer uses water as if it were a free goods, even though the price of water which he uses is determined by per unit acre of irrigated land. ③ From equation (19), we know \(0 < a'(C^*) < \infty\). Therefore, \(C^*\) is greater than zero. In other words, the farmer begins to invest in saving water.

4) In CASE II—②

The farmer’s profit function \((\pi_a)\) is

\[
(21) \quad \pi_a = P \cdot Q - P_k \cdot K - P_t \cdot L - P_b \cdot C - P_e \cdot B - Z
\]

From the necessary conditions for profit maximization, the farmer’s subjective equilibrium conditions associated with each resources’ use are

\[
(22) \quad P \cdot F_k = P_k
\]

\[
(23) \quad P \cdot F_t = P_t
\]

\[
(24) \quad P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial B} = P \cdot F_w \cdot a = P_b
\]

\[
(25) \quad P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial X} = P \cdot F_w \cdot a = Z'
\]

\[
(26) \quad P \cdot F_w \cdot \frac{\partial W}{\partial C} = P \cdot F_w \cdot A \cdot a' = P_k
\]

From equations (24), (25) and (26)

\[
(27) \quad \left| \frac{\partial W}{\partial C} \right| = \left| \frac{P_k}{P_b} \right| \quad \text{or} \quad \left| \frac{P_k}{Z'} \right|
\]

The left hand side of each equation from (22) to (27) indicates the M.V.P. of \(K\), M.V.P. of \(L\), M.V.P. of \(B\), M.V.P. of \(X\), M.V.P. of \(C\) and the M.R.S. of \(C\) to \(A\), respectively. From equations (24) and (25), the M.V.P. of \(B\) should be equal to the M.V.P. of \(X\) and these are equal to \(P_b\). \(K^{**}, L^{**}, B^{**}, X^{**}\) and \(C^{**}\), which are the optimal input of each resource respectively, are simultaneously determined by equations (22) to (26).

By the way, if we compared equations (22) to (26) to equations (8) to (11), we will notice that \(K^{**} = K^*\), \(L^{**} = L^*\), \(B^{**} = B^* + X^*\) and \(C^{**} = C^*\).

The interesting points according to these subjective equilibrium conditions compared to the results of CASE I—I are the same as that of CASE II—②.

After the Opening of a Water Market

I have already analyzed a farmer’s subjective equilibrium before the opening of a water market as it relates to four hypothetical cases. Next, I will analyze how a farmer’s subjective equilibrium will change after the opening of a water market in each case. To analyze this, I will make three additional assumptions.

(A-8): The water price per unit quantity in a water market is \(P_s\)
(A-9): The transportation costs relating to the water which the W.U.C. sells to the non-agricultural sector is zero.

(A-10): The quantity which the farmer was using before the opening of a water market is recognized as the farmer's vested right (for short, Vested Water Quantity). Therefore, the difference between the Vested Water Quantity and the water quantity (for short, \( \beta \)) which the farmer is using after the opening of a water market is recognized as the farmer's water selling quantity (for short, \( S \)), i.e., \( S = \text{Vested Water Quantity} - \beta \). And the farmer receives the amount (\( = P_s \cdot S \)) from the W.U.C.

1) In The Case Whose Initial Condition was CASE 1-1

For short, I call this case CASE 1-1'.

I assume that when the farmer buys water from the W.U.C., the price of the water is still the same as that before the opening of a water market, i.e., the water price is \( P_e \) per unit acre of irrigated land. As the farmer's Vested Water Quantity is \( B^* \), where \( B^* \) is the water quantity which the farmer was buying from the W.U.C. before the opening of a water market in CASE 1-1, \( S \) should be equal to \( (B^* - \beta) \). In order to analyze the farmer's subjective equilibrium we form the Lagrange function \( (\phi_1) \):

\[
\max_{k, l, a, s, \lambda} \phi_1 = P \cdot Q - P_k \cdot K - P_l \cdot L - P_e \cdot L - P_k \cdot C + P_e \cdot S + \lambda \cdot (B^* - \beta - S)
\]

where \( \lambda \) is the Lagrange multiplier. From necessary conditions for the maximization of the function \( (\phi_1) \), the farmer's subjective equilibrium conditions associated with each resources' use are

\[
(29) \quad P \cdot F_k = P_k
\]

\[
(30) \quad P \cdot F_i = P_i + P_e
\]

\[
(31) \quad P \cdot F_w \cdot \frac{\partial W}{\partial \beta} \cdot \frac{\partial \lambda}{\partial \beta} \equiv P \cdot F_w \cdot \alpha = P_s
\]

\[
(32) \quad P \cdot F_w \cdot \frac{\partial W}{\partial \beta} \equiv P \cdot F_w \cdot \alpha \cdot \beta = P_k
\]

From (31) and (32)

\[
(33) \quad \left| \begin{array}{c}
\frac{\partial W}{\partial \lambda} \\
\frac{\partial W}{\partial A} \\
\frac{\partial W}{\partial \beta}
\end{array} \right| = \left| \begin{array}{c}
P_k \\
P_s \\
P_k
\end{array} \right|
\]

The left hand side of each equation from (29) to (33) indicates the M.V.P. of \( K \), M.V.P. of \( L \), M.V.P. of \( \beta \), M.V.P. of \( C \) and the M.R.S. of \( C \) to \( A \), respectively. \( \tilde{K}^*, \tilde{L}^*, \beta^*, S^* \) and \( \tilde{C}^* \), which are the optimal input of each resource respectively, are simultaneously determined by equations (29) to (32) and the constraint equation.

The interesting points which arise when these subjective equilibrium conditions are compared to the results obtained in the analysis of before the opening of a water market are:

1. Even though the farmer comes to sell water at a per unit quantity price (\( P_s \)), in so far as the price of water bought from the W.U.C. is determined by per unit acre of irrigated land, land resources will be used inefficiently if compared to the perfectly competitive land market situation. This is because the farmer's M.V.P. of land is still not equal to the land market price. W is not inputted until the M.P.P. of W arrives at zero (\( \cdot \) from (31), \( F_w > 0 \)), or in other words, until total output associated with W is maximized. This result means that the farmer no longer uses water as if it were a free good.

2. The farmer buys \( \beta \) until M.V.P. of \( \beta \) becomes equal to the water market price. Consequently, water resources will be used effi-
ciently as in the perfectly competitive water market situation. This result is very interesting, because, even though the farmer buys water from the W.U.C. at a per unit acre price \( P_e \), he evaluates the water at a per unit quantity price \( P_a \) which is the water market price. \(^4\) From equation (32), we know \( 0 < \lambda'(C^*) < \infty \). Therefore, \( C^* \) is greater than zero. In other words, the farmer begins to invest in saving water.

We can think of two cases of how the farmer sells water after the opening of a water market, (a) the farmer sells some part of his Vested Water Quantity (i.e., \( B \) decreases from \( B^* \) to \( \beta \)), but he increases \( C \) to keep \( W \) at the same level as that before the opening of a water market (for short, I call this \( W \) as \( W^* \)), (b) the farmer sells some part of his Vested Water Quantity and he increases \( C \), but the increase of \( C \) is not enough to keep \( W \) at the same level as that before the opening of a water market. In other words, when associated with any level of \( \beta \), the \( C \) of case (a) [for short, \( C_a \)] is bigger than the \( C \) of case (b) [for short, \( C_b \)] and also the \( W \) of case (a) is bigger than the \( W \) of case (b).

As \( K, L, \) and \( W \) work co-operatively in the production. If the input level of \( W \) decreases, the M.V.P. curve of \( K \) and the M.V.P. curve of \( L \) will shift to the left. Therefore, the optimal input level of \( K \) and \( L \) will decrease so long as \( P_e, P_t \) and \( P_c \) are constant. The decrease of \( K \) and \( L \) will shift the M.V.P. curve of \( W \) to the left. Therefore, the M.V.P.

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Fig. 1. A farmer's water supply behavior.
curve of $W$ in case (b) is in a lower position than that of $W$ in case (a). Moreover, $\alpha(C_a)$ is bigger than $\alpha(C_b)$ \(\because C_a > C_b \). As a result, the M.V.P. curve of $\beta$ in case (b) \(= \alpha(C_b) \cdot P \cdot F_w \) is in a lower position than that of $\beta$ in case (a) \(= \alpha(C_a) \cdot P \cdot F_w \). Consequently, the optimal $\beta$ in case (b) \(\text{for short, } \beta_b^* \) is smaller than the optimal $\beta$ in case (a) \(\text{for short, } \beta_a^* \) associated with any level of $P_s$. In other words, the farmer's water selling quantity in case (b) \(\text{for short, } S_b \) is larger than that in case (a) \(\text{for short, } S_a \). This situation is demonstrated in Fig.1, where $B^*$ reflects the farmer's Vested Water Quantity. From Fig.1, the farmer's water supply behavior in case (a) is inelastic compared to that in case (b). Hence, if the farmer has an incentive to keep $W$ at the same level as that of before the opening of a water market for some reasons (for example, some anxieties about the future), the farmer's water supply behavior will be inelastic. To let the farmer behave in more elastic manner with regards to water selling to outsiders or to sell more water under the same water market price, we have to do something to help remove those anxieties.

Up to here, I have analyzed the farmer's water supply behavior in case (b) on the premise that the capital market and land market work well or that a decrease in $K$ and $L$ is very easy. But in the real world, to decrease $K$ and $L$ may not be so easy. For example, if $K$ is fixed capital, to reduce $K$ will be difficult. Also, assuming $K$ includes labor capital, it may not be easy to reduce family labor in agricultural production because of the difficulty for them to find a good job in the non-agricultural sector. Therefore, I will analyze the farmer's water supply behavior after the opening of a water market under the condition that he can not reduce $K$ and $L$ under the levels of these before the opening of a water market (i.e., $K = K^*$ and $L = L^*$). Therefore, from equations (2) and (3), $W$ will be equal to $W^*$. For short, I call this situation case (b)' Then, the M.V.P. curve of $\beta$ of case (b)' is equal to the M.V.P. curve of $\beta$ of case (a)'\(a)\). As the result of this, if there were some barriers for reducing $K$ and $L$, or the capital and land markets did not work well, the farmer's water supply behavior will be inelastic as in case (a). Therefore, to allow the farmer behave in a more elastic manner regarding the selling of water to outsiders or to sell more water under the same water market price, we have to do work with other resources' markets as well.

2) In The Case Whose Initial Condition was CASE \(\{1\} - (2)\)

For short, I call this case CASE \(\{1\} - (2)'\).

I assume that when the farmer buys water from the W.U.C., the price of the water is still the same as that before the opening of a water market, i.e., the water price is $P_h$ per unit quantity. As the farmer's Vested Water Quantity is $B^{**}$, where $B^{**}$ is the water quantity

\[ (29)' \quad P \cdot F_h[K, L; \tilde{W}^*] = P_h \]
\[ (30)' \quad P \cdot F_1[K, L; \tilde{W}^*] = P_t + P_e \]

Next, according to equations (2) and (3), let us assume $W$ is equal to $W^*$. Then $K^*$ and $L^*$ will be determined by these following two simultaneous equations.

\[ (2) \quad P \cdot F_1[K, L; W^*] = P_h \]
\[ (3) \quad P \cdot F_1[K, L; W^*] = P_t + P_e \]

From these above four equations, if $\tilde{W}^*$ is kept equal to $W^*$, $K^*$ should be equal to $K^*$ and $L^*$ should be equal to $L^*$.

Therefore, the $K$ of case (a) is equal to that of case (b)' and the $L$ of case (a) is equal to that of case (b)'. And the $C$ of case (a) is equal to that of case (b)', because the $W$ in both cases are equal to $W^*$ and the M.R.S. of $C$ to $A$ in both cases have to be equal to \[ \frac{P_h}{P_s} \]. Hence, we know that the M.V.P. curve of $\beta$ of case (b)' is equal to that of case (a).
which the farmer was buying from the W.U.C. before the opening of a water market in CASE (1) or (2), $S$ should be equal to $B^{**} - \beta$. In order to analyze the farmer's subjective equilibrium, we form the Lagrangean function ($\phi_2$):

$\max_{k,l,a,c,a',c} \phi_2 = P \cdot Q - P_k \cdot K - P_l \cdot L - P_b \cdot \beta - P_c \cdot C + P_s \cdot S + \lambda \cdot (B^{**} - \beta - S)$

where $\lambda$ is the Lagrange multiplier. From the necessary conditions for the maximization of the function ($\phi_2$), the farmer's subjective equilibrium conditions associated with each resources' use are

(35) $P \cdot F_k = P_k$

(36) $P \cdot F_l = P_l$

(37) $P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial \beta} = P \cdot F_w \cdot a = P_b + P_s$

(38) $P \cdot F_w \cdot \frac{\partial W}{\partial C} = P \cdot F_w \cdot a' \cdot \beta = P_k$

From equations (37) and (38)

(39) $\left| \frac{\partial W}{\partial C} \right| = \left| \frac{P_k}{P_b + P_s} \right|$

The left hand side of each equation from (35) to (39) represents the M.V.P. of $K$, M.V.P. of $L$, M.V.P. of $\beta$, M.V.P. of $C$ and the M.R.S. of $C$ to $A$, respectively. $K^{**}$, $L^{**}$, $\beta^{**}$, $S^{**}$ and $C^{**}$, which are the optimal input of each resource respectively, are simultaneously determined by equations (35) to (38) and the constraint equation.

The interesting point which arise when these subjective equilibrium conditions are compared to the results obtained in the analysis of before the opening of a water market is that the price of water bought from the W.U.C. which the farmer thinks of subjectively (for short, Farmer's Subjective Water Price) is not $P_s$, but $(P_b + P_s)$. That is to say, the farmer comes to evaluate the per unit quantity of water bought from the W.U.C. as $(P_b + P_s)$ which is higher than the water market price $(P_s)$. In other words, the farmer starts to think that the water which he uses is more valuable than the water as evaluated by the water market price. Consequently, the farmer begins to use too little water compared to the perfectly competitive water market situation, because the Farmer's Subjective Water Price (or the M.V.P. of $\beta$ on his subjective equilibrium) is higher than the water market price. Therefore, water resources will be used inefficiently compared to the perfectly competitive water market situation.

After the opening of a water market, the total available water resources will be allocated between in the agricultural sector and in the non-agricultural sector as demonstrated in Fig. 2. In Fig. 2, $\text{WAT}$ is the total available water resources, $\text{WAT}_a$ is the quantity of water resources which will be used in the agricultural sector and $\text{WAT}_n$ is the quantity of water resources which will be used in the non-agricultural sector. Thus, after the opening of a water market, the gap (for short, $\text{GAP}_1$) between the M.V.P. of water in the agricultural sector and that in the non-agricultural sector will be $P_b$. On the other hand, in the situation before the opening of a water market, from equation (10), the M.V.P. of water in the agricultural sector is $P_b$. And, for convenience, let the M.V.P. of water in the non-agricultural sector be $P_n$. Therefore, before the opening of a water market, the gap (for short, $\text{GAP}_1$) between the M.V.P. of water in the agricultural sector and that in the non-agricultural sector is $(P_n - P_b)$. If $\text{GAP}_2$ is greater than $\text{GAP}_1$, even though water resources will still be used
inefficiently after the opening of a water market (compared to the perfectly competitive water market situation), we will be able to say that water resources will be used more efficiently when we compare this water resources' use situation to that of before the opening of a water market. But, if GAP₂ is smaller than GAP₁, the above mentioned conclusion will be opposite.

The interesting point when we compare the results obtained here to the results obtained in CASE [II-1] is that even though the water market was opened, if the farmer buys water according to a per unit acre price from the W.U.C., water resources are used efficiently but land resources come to be used inefficiently. If the farmer buys water at a per unit quantity price from the W.U.C., water resources will be used inefficiently but land resources come to be used efficiently. Therefore, from the standpoint of minimizing social losses associated with various resources' use, we can not say anything about which system is better until we get more information about production functions and the price of each resources.

3) In The Case Whose Initial Condition was [II-1]’

For short, I call this case CASE [II-1]’.

I assume that when the farmer buys water from the W.U.C., the price of the water is still the same as that before the opening of a water market, i.e., the water price is \( P_a \) per unit acre of irrigated land. As the farmer's Vested Water Quantity is \( B^* \), where \( B^* \) is the water quantity which the farmer was buying from the W.U.C. before the opening of a water market in CASE [II-1], \( S \) should be equal to \( (B^* - \beta) \). In order to analyze the farmer’s subjective equilibrium, we form the Lagrangean function \( \phi_3 \):
\[
\begin{align*}
\text{(40)} & \quad \max_{k,l,d,x,s,c} \phi = P \cdot Q - P_k \cdot K - P_t \cdot L - P_e \cdot L_1 - P_c \cdot C - Z + P_s \cdot S + \lambda \cdot (B^* - S - \beta) \\
\text{(41)} & \quad P \cdot F_k = P_k \\
\text{(42)} & \quad P \cdot F_l = P_t + P_e \cdot \frac{\beta}{A} \\
\text{(43)} & \quad P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial \beta} = P \cdot F_w \cdot a = P_e \cdot \frac{L}{A} \cdot \frac{X}{A} + P_s \\
\text{(44)} & \quad P \cdot F_w \cdot \frac{\partial W}{\partial A} \cdot \frac{\partial A}{\partial \beta} = P \cdot F_w \cdot a = Z' - P_e \cdot \frac{L}{A} \cdot \frac{\beta}{A} \\
\text{(45)} & \quad P \cdot F_w \cdot \frac{\partial W}{\partial c} \equiv P \cdot F_w \cdot a \cdot A = P_k \\
\end{align*}
\]

From equations (43), (44) and (45)

\[
\begin{align*}
\frac{\partial W}{\partial A} & = \left| \begin{array}{c}
P_e \\
P_e \cdot \frac{L}{A} \cdot \frac{X}{A} + P_s \\
Z' - P_e \cdot \frac{L}{A} \cdot \frac{\beta}{A}
\end{array} \right| \\
\text{or} & = \left| \begin{array}{c}
P_k \\
Z' - P_e \cdot \frac{L}{A} \cdot \frac{\beta}{A}
\end{array} \right|
\end{align*}
\]

The left hand side of each equation from (41) to (46) represents the M.V.P. of K, M.V.P. of L, M.V.P. of \( \beta \), M.V.P. of \( X \), M.V.P. of \( C \) and the M.R.S. of \( C \) to \( A \), respectively. From (43) and (44), the M.V.P. of \( \beta \) should be equal to the M.V.P. of \( X \). \( \hat{K}^*, \hat{L}^*, \hat{\beta}^*, S^*, \hat{X}^* \) and \( \hat{C}^* \), which are the optimal input of each resource respectively, are simultaneously determined by equations (41) to (45) and the constraint equation.

The interesting points which arise when these subjective equilibrium conditions are compared to the results obtained in the analysis of before the opening of a water market are: 1) even though the farmer is able to sell water at a per unit quantity price \( P_x \), in so far as the price of water bought from the W.U.C. is determined by per unit acre of irrigated land, land resources will still be used inefficiently when we compare this land resources' use situation to the perfectly competitive land market situation. Because, the farmer's M.V.P. of land on his subjective equilibrium is still not equal to the land market price. 2) Farmer's Subjective Water Price is not \( P_e \cdot \frac{L}{A} \cdot \frac{X}{A} \) (marginal increase of payment to the W.U.C.), but \( P_e \cdot \frac{L}{A} \cdot \frac{X}{A} + P_s \).

That is to say, the farmer evaluates the per unit quantity of water bought from the W.U.C. as \( P_e \cdot \frac{L}{A} \cdot \frac{X}{A} + P_s \) which is higher than the water market price \( P_x \). In other words, the farmer starts to think that the water which he uses is more valuable than the water as evaluated by the water market price. Consequently, the farmer comes to use too little water when we compare this to the perfectly competitive water market situation, because the Farmer's Subjective Water Price (or the M.V.P. of \( \beta \) on his subjective equilibrium) is higher than the water market price. Therefore, when we compare this water resources' use situation to the perfectly competitive water market situation, water resources will be used inefficiently. 3) From equations (43) and (44), as the marginal pumping costs on \( \hat{X}^* \) is greater than the water market price \( P_s \), when we compare this underground water use situation to that in the case of perfectly competitive water market situation, underground water will be overdrafted.

If the water market in the non-agricultural sector is perfectly competitive, after the opening of a water market, the M.V.P. of water in the non-agricultural sector will be equal to \( P_s \). Therefore, after the opening of a water market, the gap (for short, GAP\(_3\)) between the M.V.P. of water in the agricultural sector and that in the non-agricultural sector will be
On the other hand, in the situation before the opening of a water market, from equation (17), the M.V.P. of water in the agricultural sector is \( P_e \cdot \frac{L}{A} \cdot \frac{X}{A} \). And, for convenience, let the M.V.P. of water in the non-agricultural sector be \( P_n \). Therefore, before the opening of a water market, the gap (for short, GAP), between the M.V.P. of water in the agricultural sector and that in the non-agricultural sector is \( P_n - P_e \cdot \frac{L}{A} \cdot \frac{X}{A} \). If GAP is greater than GAP, even though water resources will still be used inefficiently after the opening of a water market (compared to the perfectly competitive water market situation), we will be able to say that water resources will be used more efficiently when we compare this water resources' use situation to that of before the opening of a water market. But, if GAP is smaller than GAP, the above mentioned conclusion will be opposite.

4) In The Case Whose Initial Condition was CASE (II) - (2).

For short, I call this case CASE (II) - (2). I assume that when the farmer buys water from the W.U.C., the price of the water is still the same as that before the opening of a water market, i.e., the water price is \( P_b \) per unit quantity. As the farmer's Vested Water Quantity is \( B^* \), where \( B^* \) is the water quantity which the farmer was buying from the W.U.C. before the opening of a water market in CASE (II) - (2), \( S \) should be equal to \( (B^* - \beta) \). In order to analyze the farmer's subjective equilibrium, we form the Lagrangean function \( (\phi_i) \):

\[
\max_{k, l, \alpha, s, c} \phi_i = P \cdot Q - P_k \cdot K - P_l \cdot L - P_b \cdot \beta - P_h \cdot C - Z + P_s \cdot S + \lambda \cdot (B^* - S - \beta)
\]

where \( \lambda \) is the Lagrange multiplier. From necessary conditions for the maximization of the function \( (\phi_i) \), the farmer's subjective equilibrium conditions associated with each resources' use are

\[
P \cdot F_k = P_k
\]

\[
P \cdot F_l = P_l
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial A} - \frac{\partial A}{\partial \beta} \equiv P \cdot F_w \cdot a = P_b + P_s
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial A} - \frac{\partial A}{\partial \lambda} \equiv P \cdot F_w \cdot a = Z
\]

\[
P \cdot F_w \cdot \frac{\partial W}{\partial C} \equiv P \cdot F_w \cdot a \cdot A = P_k
\]

From equations (50), (51) and (52)

\[
\begin{vmatrix}
\frac{\partial W}{\partial C} \\
\frac{\partial W}{\partial A}
\end{vmatrix} = \begin{vmatrix}
P_k \\
P_b + P_s
\end{vmatrix} \quad \text{or} \quad \begin{vmatrix}
P_k \\
Z
\end{vmatrix}
\]

The left hand side of each equation from (48) to (53) represents the M.V.P. of \( K \), M.V.P. of \( L \), M.V.P. of \( \beta \), M.V.P. of \( X \), M.V.P. of \( C \) and the M.R.S. of \( C \) to \( A \), respectively. From equations (50) and (51), the M.V.P. of \( \beta \) should be equal to the M.V.P. of \( X \). \( K^*, L^*, \beta^*, S^*, X^*, C^* \), which are the optimal input of each resource respectively, are simultaneously determined by equations (48) to (52) and the constraint equation.

The interesting points which arise when these subjective equilibrium conditions are compared to the results obtained in the analysis of before the opening of a water market are the same in
CASE $\mathbb{I}-\mathbb{2}'$. Another additional interesting point is that from equations (50) and (51), as the marginal pumping costs on $\bar{X}^{**}$ is greater than the water market price $P_w$, underground water will be overdrafted when we compare this underground water use situation to that in the case of perfectly competitive water market situation.

**Discussion**

To use water resources efficiently, it becomes necessary to introduce a water market which connects the agricultural sector with the non-agricultural sector. Even so, to introduce such a water market is not so easy and how we introduce this water market is a very important problem.

In Japan, from two thousand years ago up until several decades ago, almost all water resources had been used for agricultural production (especially, for rice production). Therefore, for farmers, the introduction of a water market which connects the agricultural sector and the non-agricultural sector will lead to drastic changes or they may feel that they are under a water market revolution.

If the introduction of a water market presents some big changes to farmers, they will not be able to anticipate what kinds of merits and demerits they will obtain from its introduction. As farmers have a strong preference for risk aversion, they will be more concerned with demerits than with merits. If they can not anticipate the demerits precisely, they will be nervous about the introduction of a water market and as a result, they will have a strong incentive to block this introduction by using their strong political power.

If the plan to introduce a water market is cancelled, to restart discussion about the introduction of a water market, it will be necessary to wait for a long time until farmers’ anger becomes cooled down. Therefore, to introduce a water market smoothly, it would be better to introduce it step by step without drastic changes, so that farmers can anticipate the merits and demerits caused by the introduction. When we introduce a water market, from the first step, to persist in pursuing 100% efficient workability of such a water market which will cause drastic changes will not be so efficient, if the introduction of a water market is blocked by farmers and if we have to wait a long time to re-discuss it.

Therefore, for the smooth introduction of a water market, it is best to introduce a water market which allows farmers to continue to hold their vested rights associated with water resources’ use which they have held, i.e., to allow farmers to be able to sell water while keeping their water rights.

In this paper, it has been seen that when a water market is introduced while allowing farmers to maintain their vested rights, a farmer’s subjective equilibrium associated with each resources’ use and the efficiency of each resources’ use will change as follows:

1. In CASE $\mathbb{I}-\mathbb{1}'$, water resources will be used efficiently as in the perfectly competitive water market situation. But, when we compare the land resources’ use situation obtained here to the perfectly competitive land market situation, land resources will be used inefficiently. Even so, the level of inefficiency of use of land resources is still the same as that of before the opening of a water market. Therefore, from the standpoint of minimizing social losses associated with each resources’ use, we can say that introduction of a water market is beneficial to society.

2. In each case of CASE $\mathbb{I}-\mathbb{2}'$, CASE $\mathbb{II}-\mathbb{1}'$ and CASE $\mathbb{II}-\mathbb{2}'$, when we compare the water resources’ use situation to the perfectly competitive water market situation, water resources will be used inefficiently. Even so, after the opening of a water market, if the gap between the M.V.P. of water in the agricultural sector and the M.V.P. of water in the non-agricultural sector is smaller than that in the case of before the opening of a water market, we will be able to say that water resources will be used more efficiently than the water resources use situation before the opening of a water market. But, the opposite
situation of those gaps leads to the opposite conclusion. 

(3) The land resources’ use situation in CASE Ⅱ−①’ will remain inefficient when we compare this land resources’ use situation to the perfectly competitive land market situation. But, the land resources’ use in CASE Ⅰ−②’ and CASE Ⅱ−②’ will still be efficient as in the perfectly competitive land market situation.

(4) After the opening of a water market, from the standpoint of minimizing social losses associated with each resources’ use, we can not say anything about which way of water pricing by the price per unit acre of irrigated land or the price per unit quantity is better, until we get more information about production functions and the price of each resources.

(5) For some anxieties in the future, if farmers have the incentive to keep \( W \), which is the quantity of water that is actually available for the plant in the root zone, at the same level as that of before the opening of a water market or if the other resources’ markets do not work well, farmer’s water supply behavior will be inelastic. Therefore, when we think about efficient water resources’ use, we have to pay much attention to other resources’ markets’ situations, too.

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水市場導入と農家の主体均衡

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水資源を効率的に利用するためには、農業セクターと非農業セクターを結びつけることの出来る水市場の導入が必要となってくる。しかしながら、このような水市場の導入は容易ではなく、しかもどのようにして水市場を導入するか大きな課題である。とくに多量の水利権をすでに保有している農業セクターにおいては、このような憑まれた水利用の既得権を失なう恐れのあるような形での水市場の導入に対しては反対するであろう。もしそうならば水市場の導入は出来ないことになってしまう。

そこで本論文では、農業セクターに水利権利用の既得権を認めるような形で水市場導入を行なったとき、水資源利用および水資源組み合わされて農業生産に利用される資源（たとえば土地）の利用に関して、農家の主体均衡はどのように変化し、その結果として、国民経済的にみた場合これら資源利用の効率性はどのように変化するのかについて検討を行なう。

農業セクターに対し水利権の既得権を認められるような限定された形での水市場の導入であっても、一般的に言って、水市場の導入によって水資源利用の効率性をアップすることが出来る可能性がある。ただし水市場を導入するにあたって、どのような水価格政策を実施するかによって、水資源の効率的な利用を達成出来ない場合も発生する。さらにその水価格政策の影響として、水と組み合わされて農業生産に利用される他の資源（たとえば土地）の効率的利用も達成出来なくなる可能性もある。したがって水市場を導入するにあたっては、いかなる水価格政策を実施するのかについても慎重な検討がなされなければならない。